

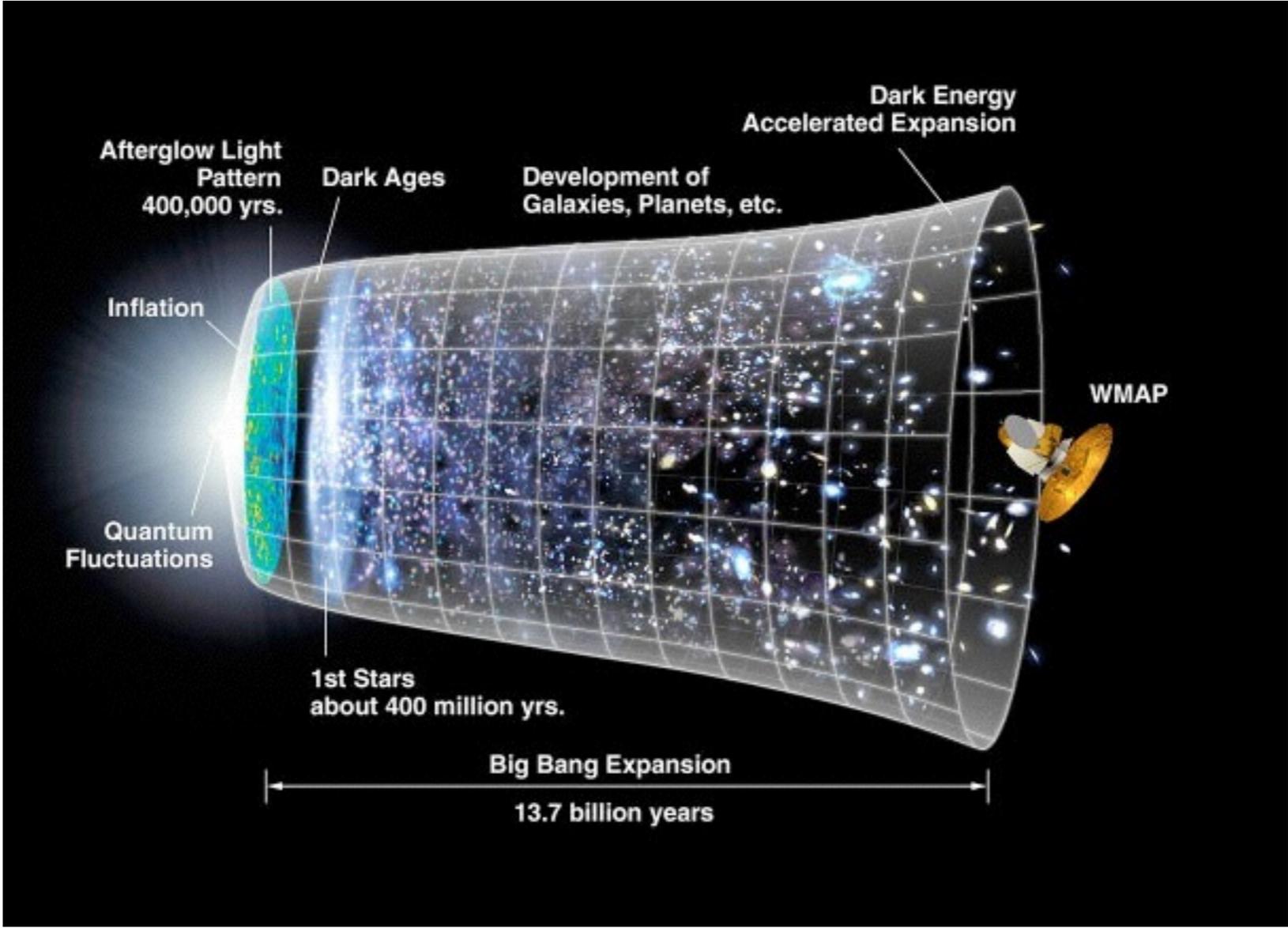
The Accelerating Universe

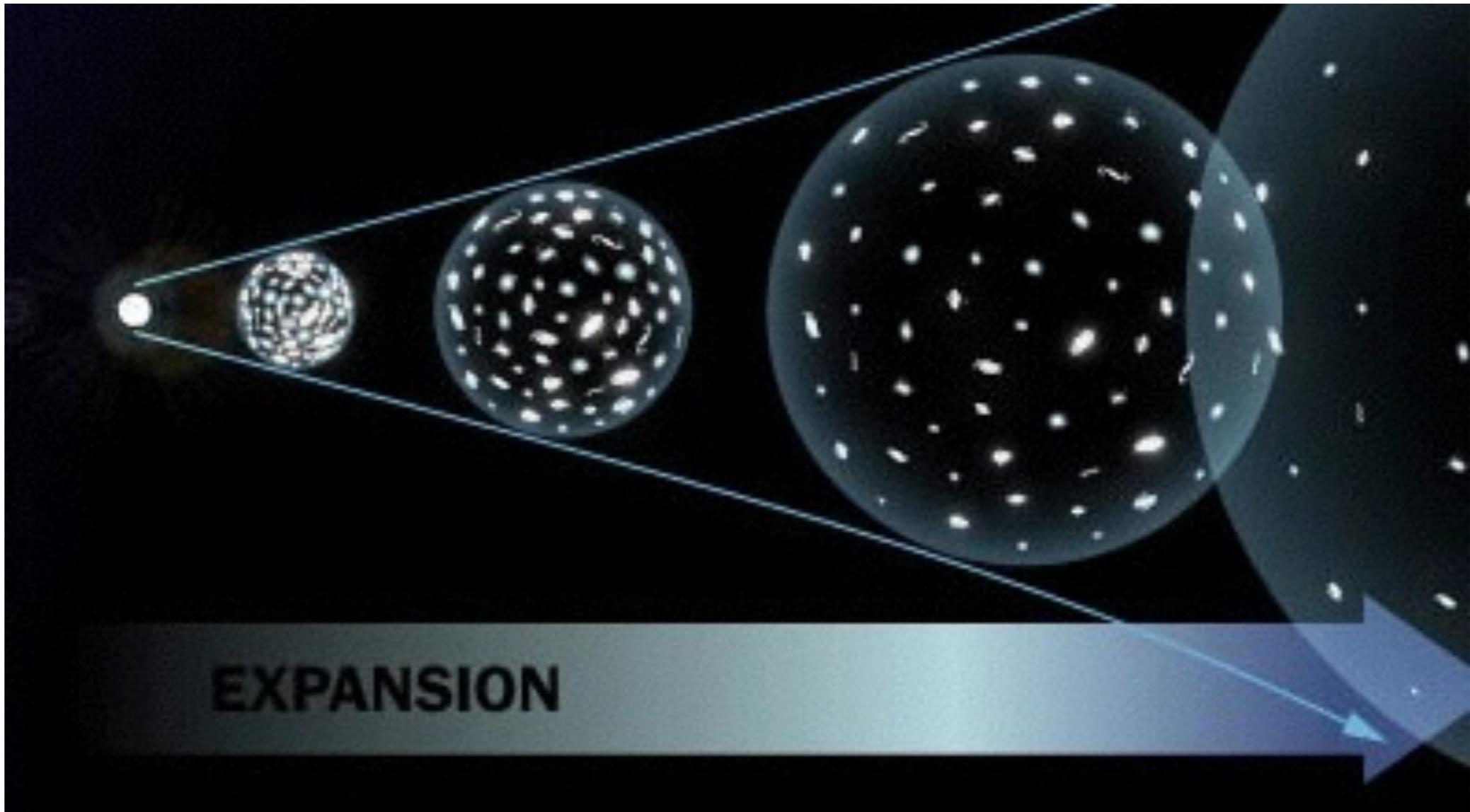
Anne Davis, DAMTP

What do I do? I'm a mathematical physicist and use the physics of the very small to explain phenomena of the very large!

Outline

- Introduction to Cosmology
- The Accelerating Universe
- Possible Explanations
- Chameleon Theories
- A Neat Application to Black Holes
- Conclusions





Cosmology

work in natural units

$$c = k_B = \hbar = 1$$

$$1\text{Gev} = 1.78 \times 10^{-24}g; 1\text{Gev}^4 = 2.32 \times 10^{17}g/cm^3$$

the critical density

$$\rho_{crit} = 10^{-29}g/cm^3 = 4 \times 10^{-47}\text{Gev}^4$$

one usually writes the space vector as

$$\mathbf{x}^2 = x^2 + y^2 + z^2$$

Now consider space-time

$$s^2 = -t^2 + \mathbf{x}^2 = -t^2 + x^2 + y^2 + z^2$$

In Cosmology we go a bit further and consider

$$ds^2 = -dt^2 + a(t)^2 dx^2$$

this generalises the invariant distance from special relativity; $a(t)$ is called the scale factor of the universe. This metric governs early universe cosmology and is called the FRW metric.

Here we have rescaled distances by the scale factor $a(t)$ of the Universe

$$x = a(t)x_0$$

so we factor out the expansion of the universe, where \mathcal{X} is the proper distance and \mathcal{X}_0 is the comoving distance. This enables us to derive the basic equations, the Friedmann equation in terms of $a(t)$.

This relates the expansion of the Universe to the properties of matter in the Universe. There is a relation between pressure and density called the equation of state. Using this we solve the equation for different sorts of matter in the Universe.

Usually we derive the equations governing cosmology from Einstein equations of General Relativity, but they can be derived just from Newton's Laws and the metric (Part II).

These equations are a set of coupled differential equations, similar to those of IA!

$$\frac{\dot{a}^2}{a} = H^2 = \frac{8\pi G}{3}\rho + \Lambda/3$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho + 3p) + \Lambda/3$$

and energy conservation gives

$$\dot{\rho} = -3H(\rho + p)$$

where ρ is the energy density and p the pressure

These simple equations govern the behaviour of the universe.

To solve them take

$$p = \omega \rho$$

giving $\rho = a^{-3(1+\omega)}$

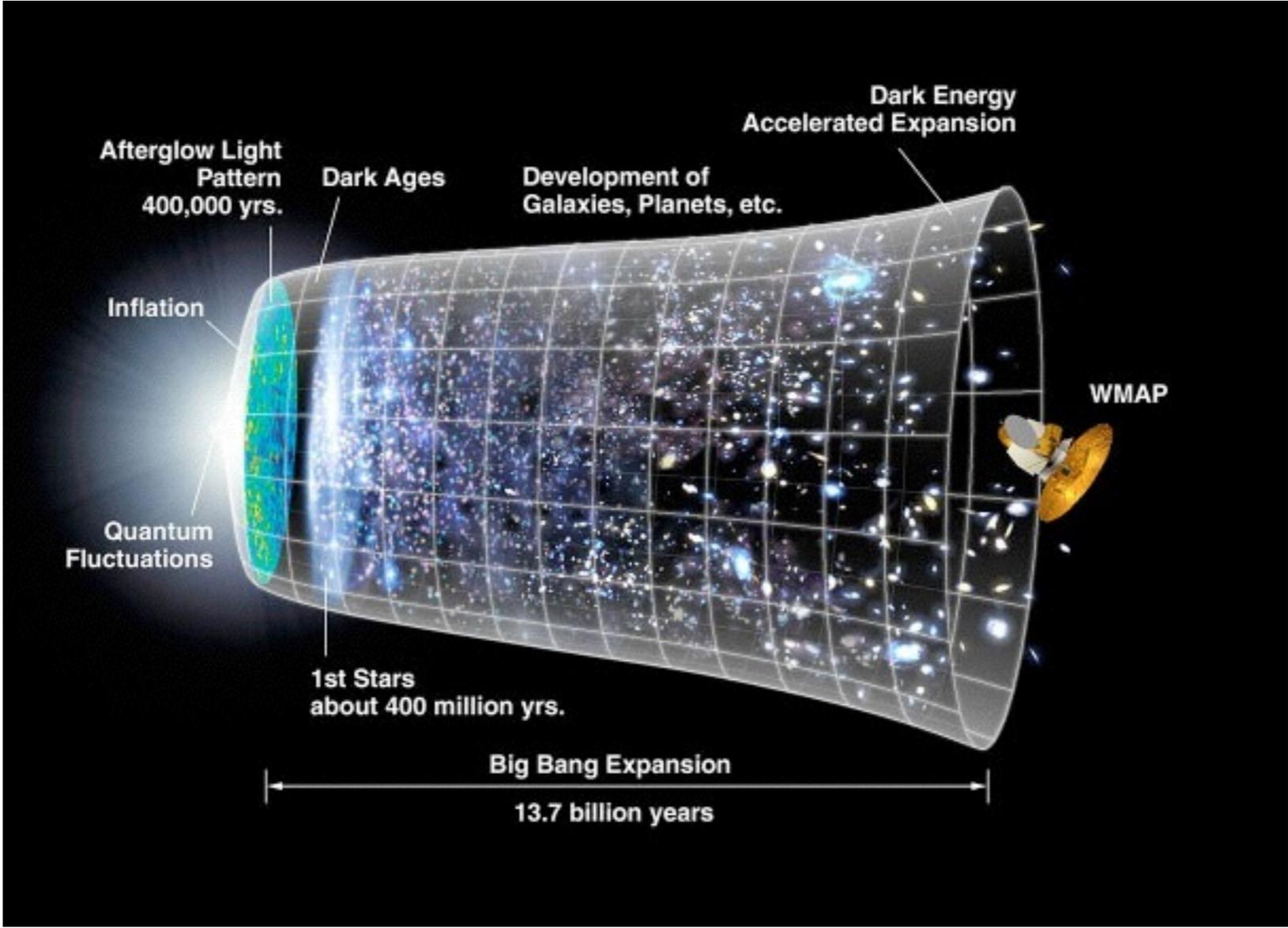
We find solutions depending on ω

matter $\omega = 0, a(t) \approx t^{2/3}$

radiation $\omega = 1/3, a(t) \approx t^{1/2}$

cosmological constant $\omega = -1, a(t) \approx \exp \Lambda t$

our knowledge of the universe is inferred from a variety of experiments.



The critical density is

$$\rho_c = 3H^2 / 8\pi G$$

and the density parameter

$$\Omega = \rho / \rho_c$$

this determines whether the universe is flat, open or closed.

Currently we think the universe is flat and Ω is one.

$$\Omega = \Omega_{Tot} = \Omega_m + \Omega_{DE}$$

$$\Omega_{DE} \equiv \Omega_\Lambda$$

How much matter there is in the
Universe determines its ultimate fate

Friedmann equation $H^2 = \frac{8\pi G}{3} \rho$

The critical density is $\rho_c = 3H^2 / 8\pi G$

and the density parameter $\Omega = \rho / \rho_c$

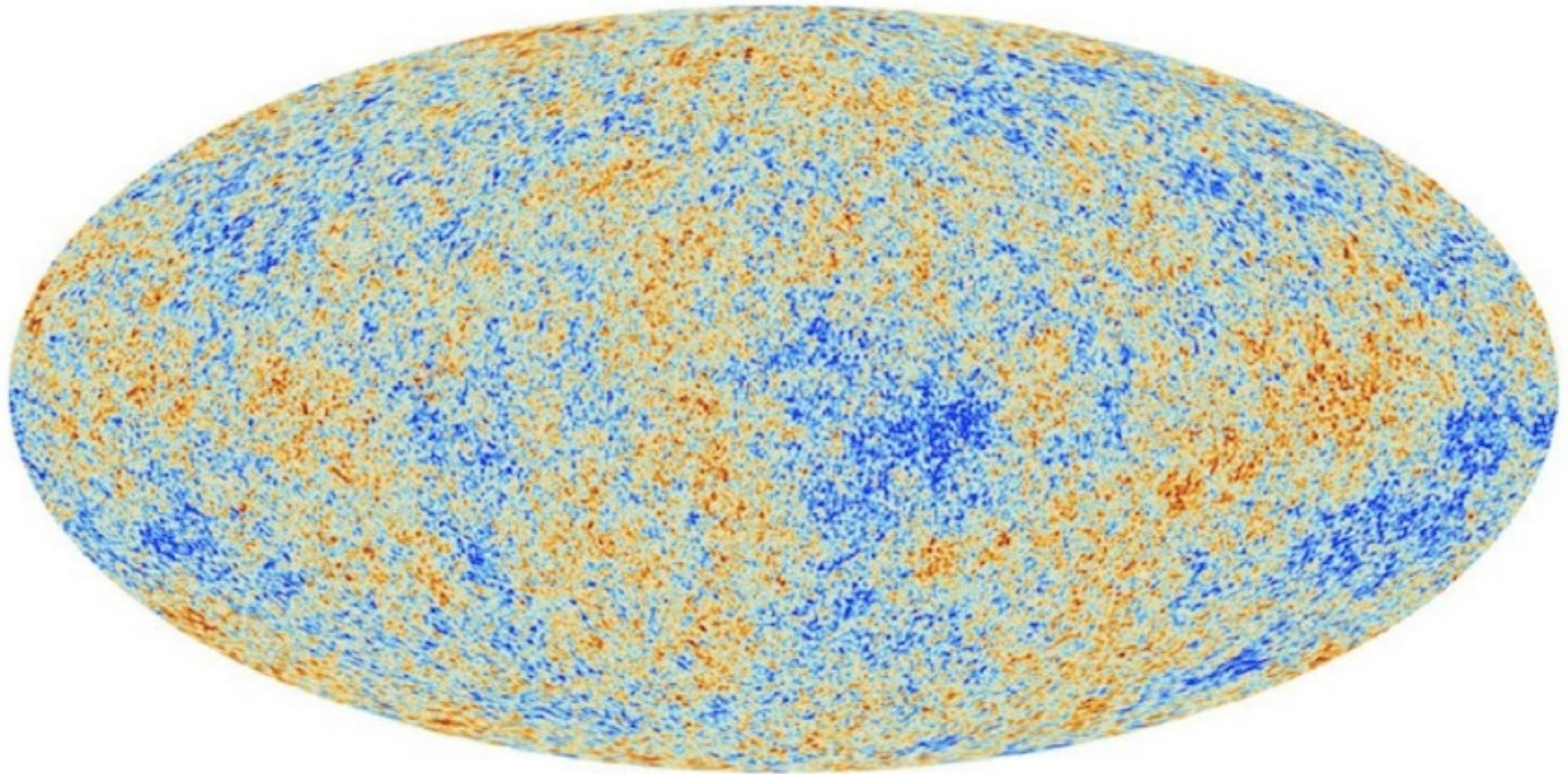
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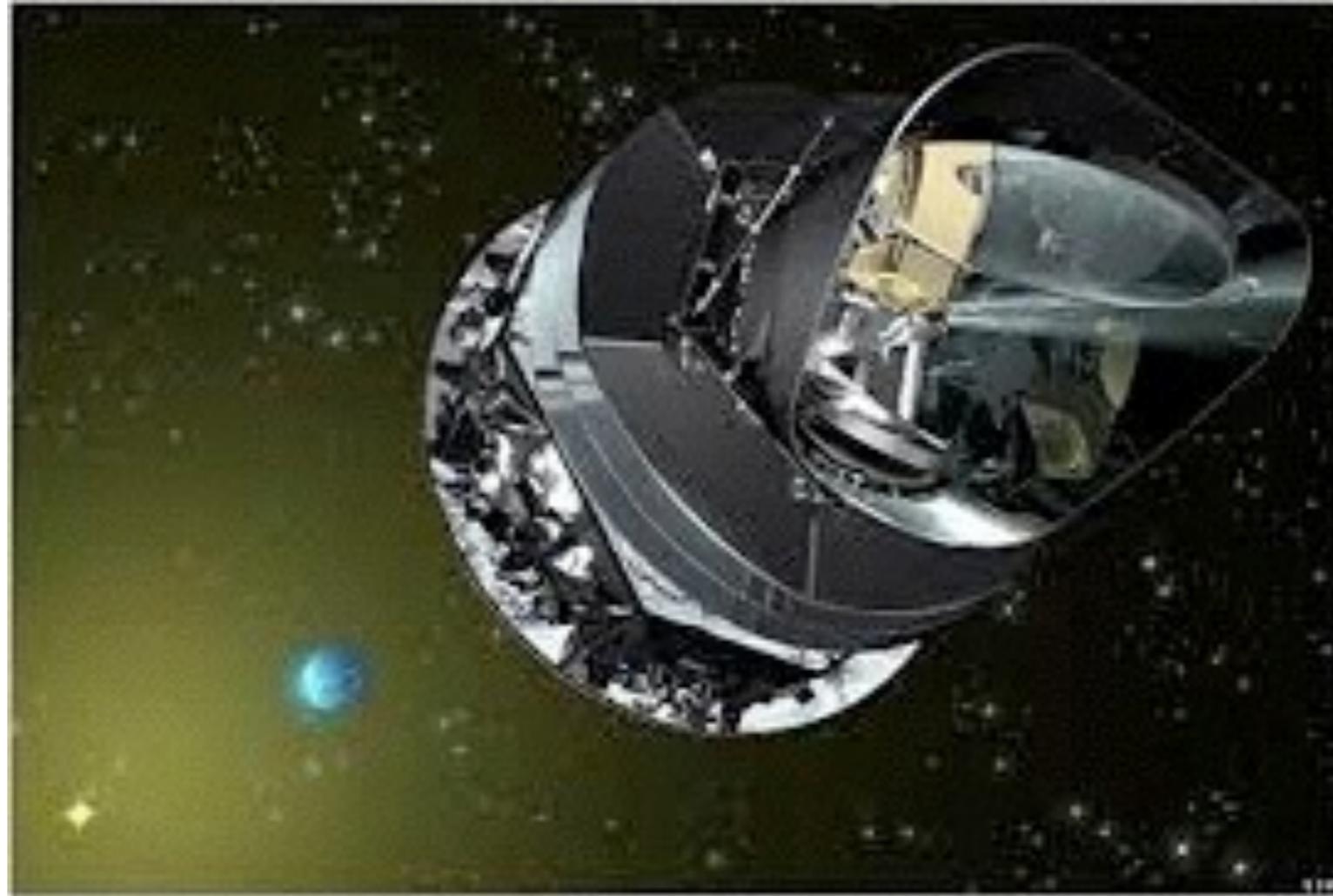
$$\Omega = \Omega_{Tot} = \Omega_m + \Omega_{DE}$$

How?

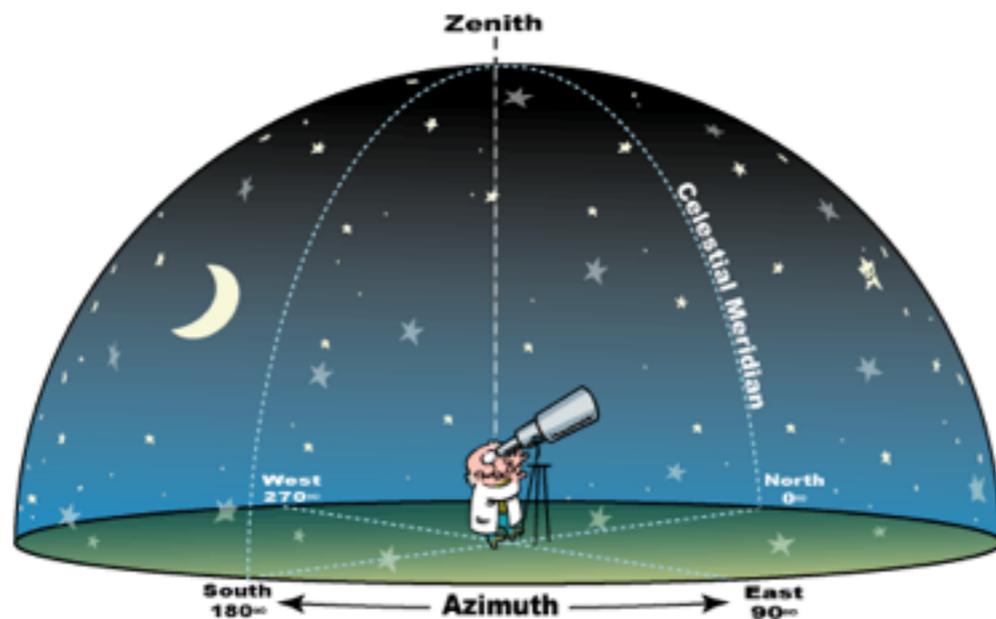
Cosmic Microwave Background



Planck Satellite



Supernova Experiments



Absolute luminosity.

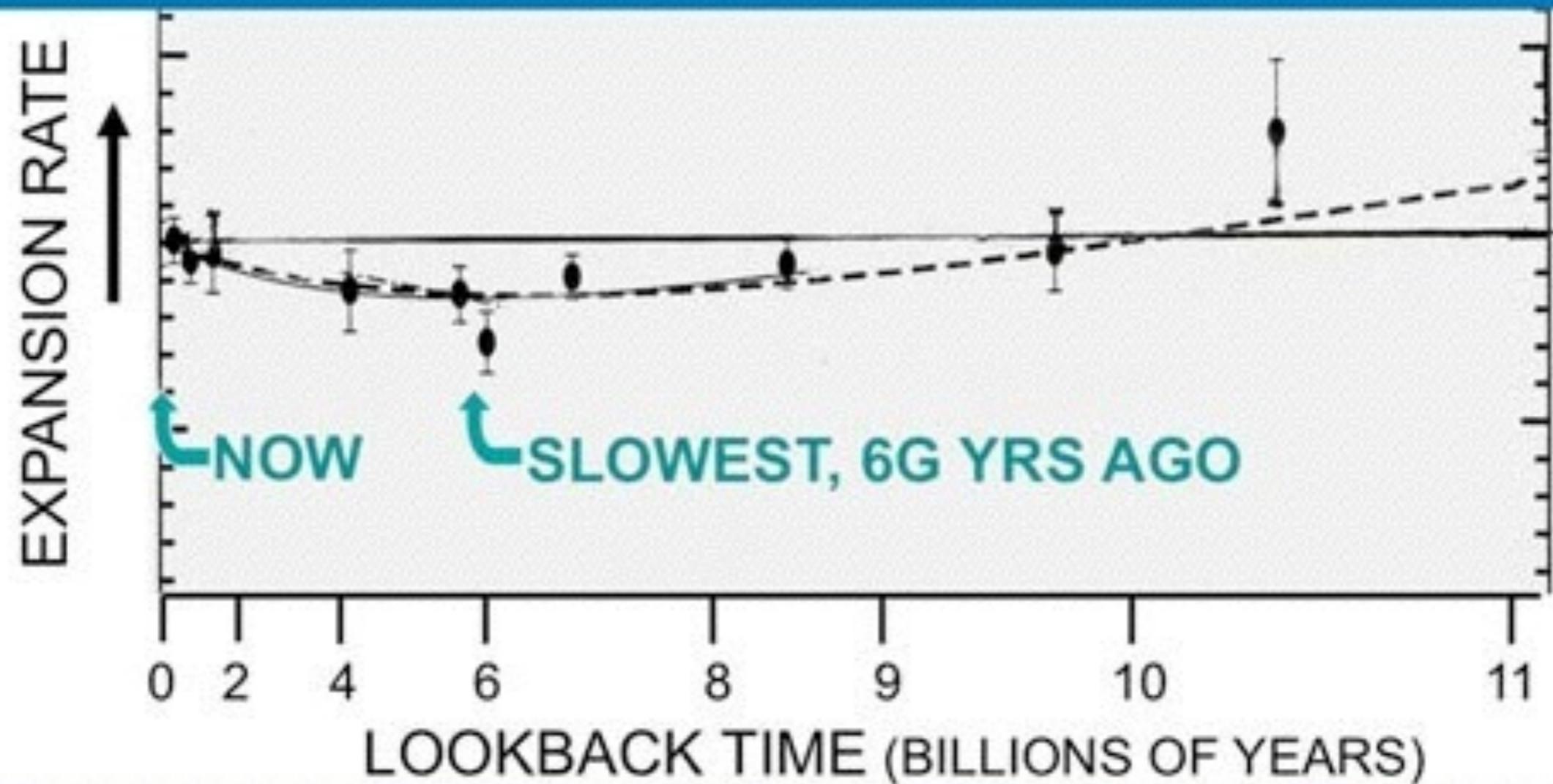
$$d_L = \sqrt{L/4\pi\Phi_R} = c/H_0[z + 1/2(1 - q_0)z^2 + ..]$$

Received flux: what
we see in the telescope ...

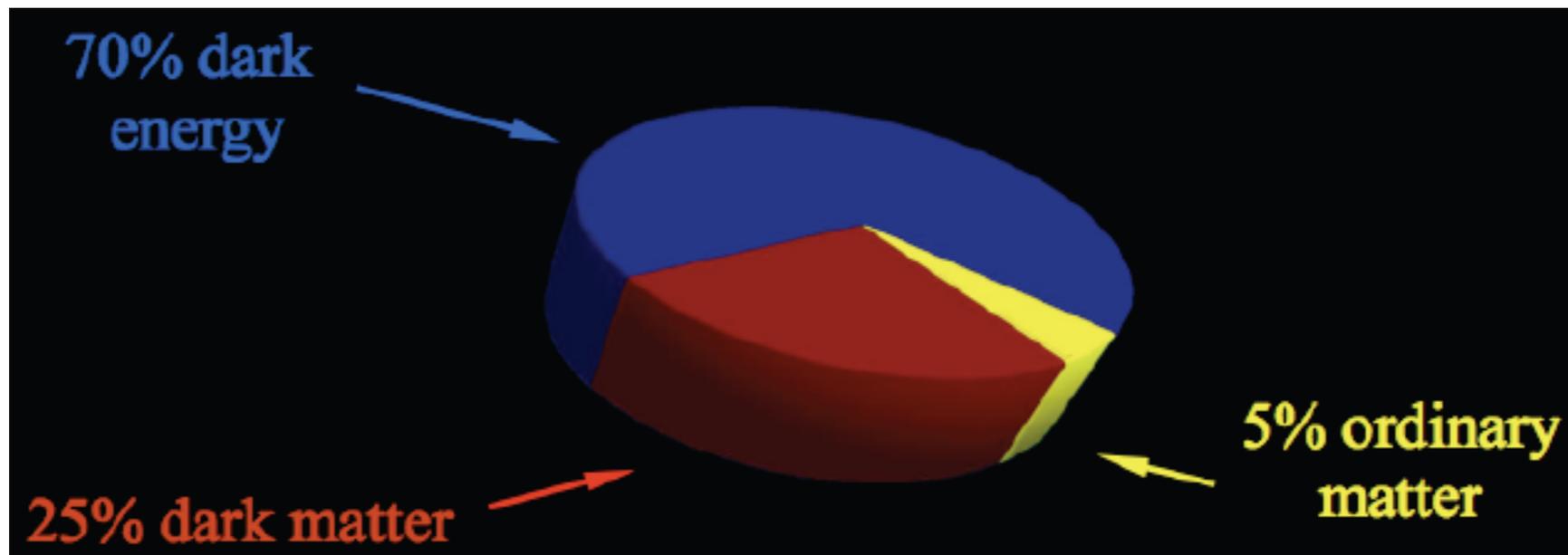
acceleration parameter:
we need large red-shift z

$$\frac{a}{a_0} = \frac{1}{1+z}$$

Expansion is Accelerating



This gives us the energy budget of the universe

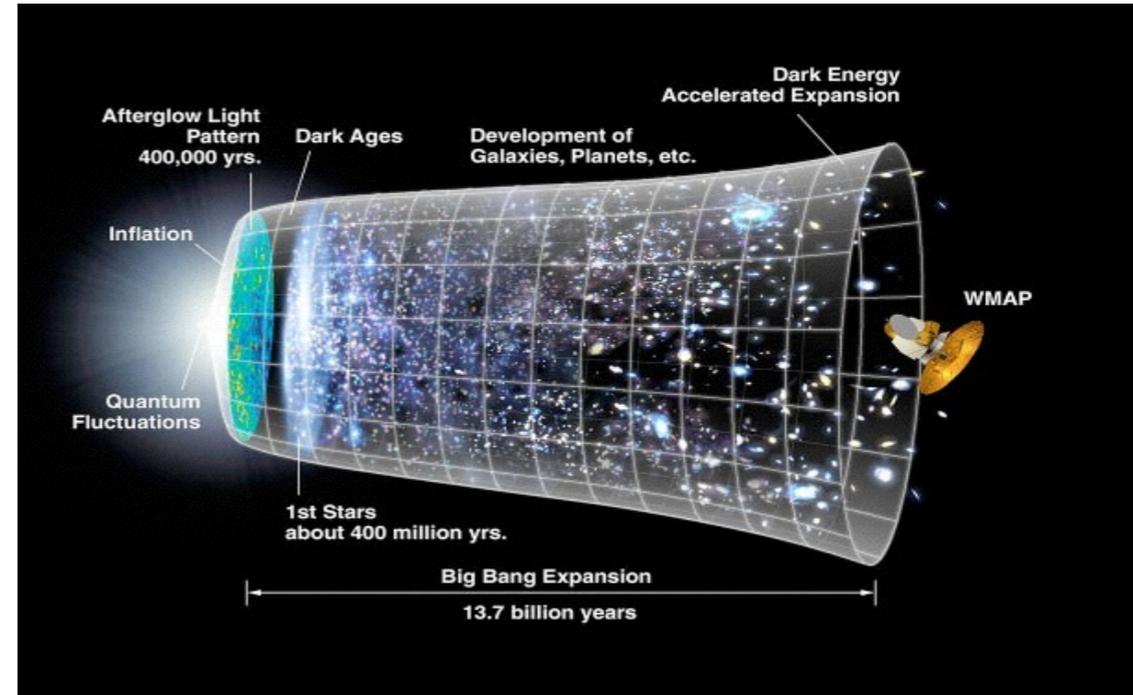


What is dark energy?

Phases of the Universe



Shelly-Ann Fraser-Pryce



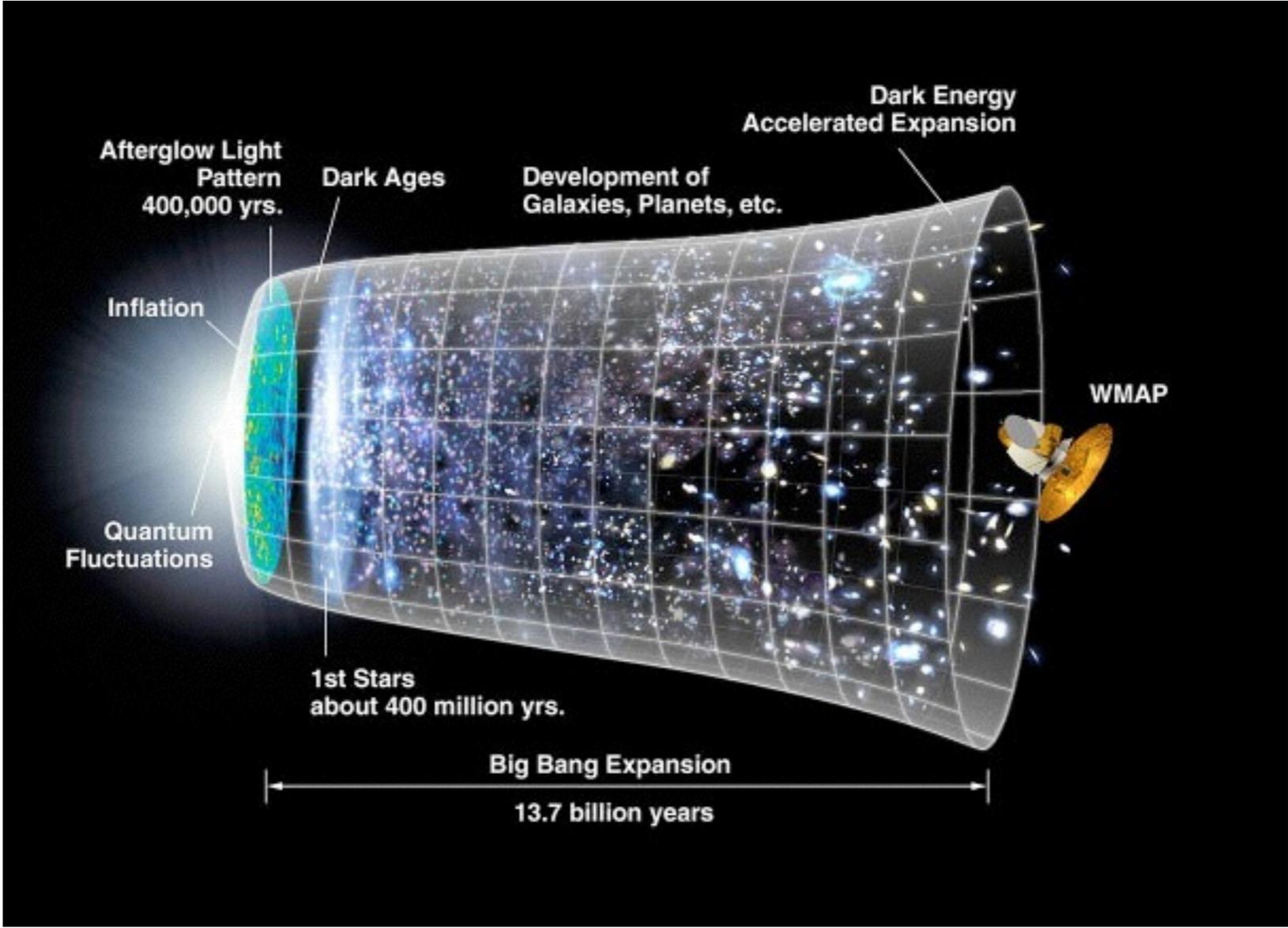
Christine Ohuruogu



Kelly Holmes



Paula Radcliffe



What is Dark Energy?

It could be the cosmological constant

$$\Lambda$$

But it could be due to the dynamics of a scalar field

$$\phi(\mathbf{x}, t)$$

Something which has a value at every point in space and time but has no favoured direction

Write down the Action and calculate the equations of motion and properties

For a scalar field

$$\rho_\phi = 1/2\dot{\phi}^2 + V(\phi) \quad \text{kinetic energy + potential energy}$$

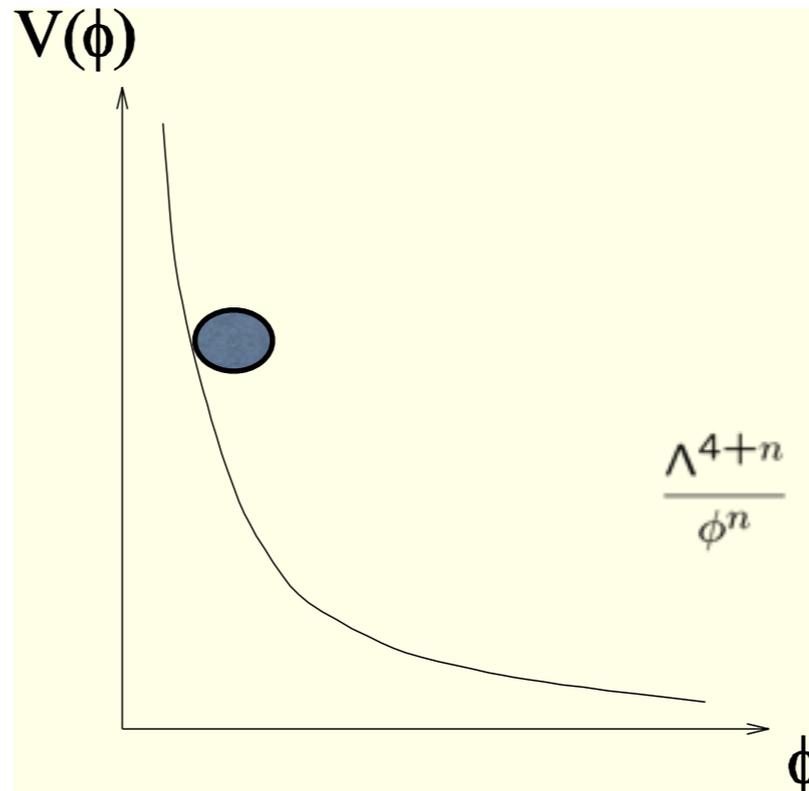
$$p_\phi = 1/2\dot{\phi}^2 - V(\phi) \quad \text{kinetic energy - potential energy}$$

If the potential dominates then

$$p_\phi \approx -\rho_\phi \quad \text{so} \quad \omega \approx -1$$

so the scalar field plays the role of an effective cosmological constant. Since it's dynamical, this wouldn't have been the case for all times in the universe. We only need the scalar field to dominate the energy density of the universe today

This modifies our theory to



Field rolling down a runaway potential, reaching large values now.

If the field couples to ordinary matter
it will give rise to a fifth force

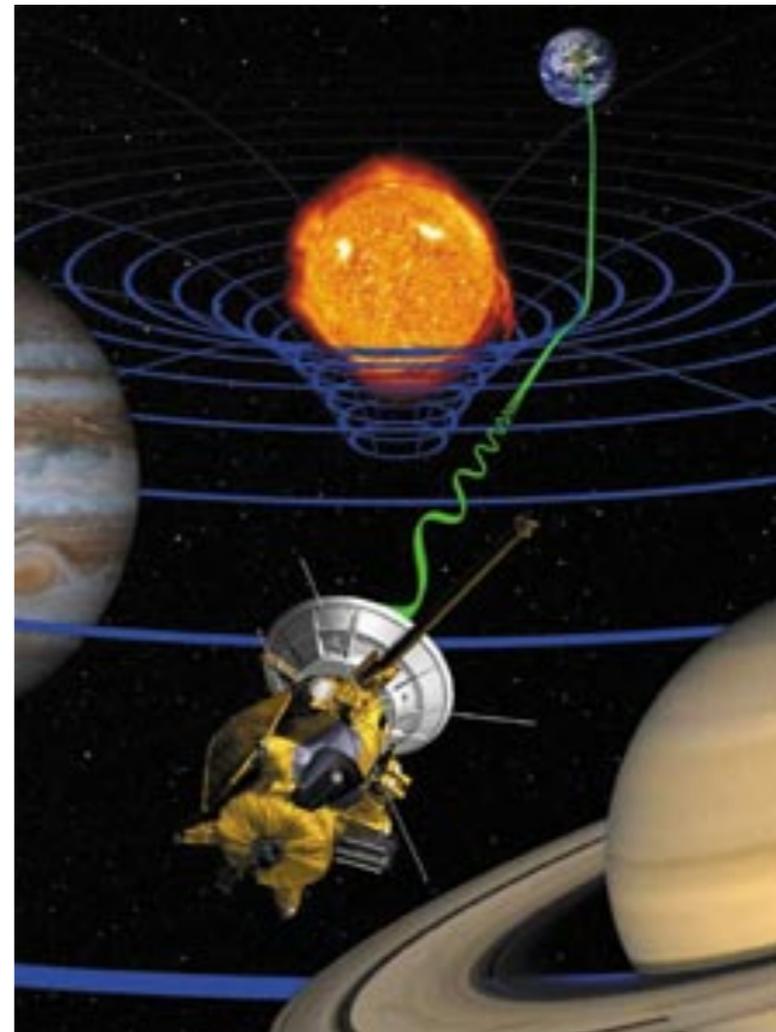
Deviations from Newton's
Laws parametrised by

$$\Phi_N = -G_N/r(1 + 2\beta^2 e^{-r/\lambda})$$

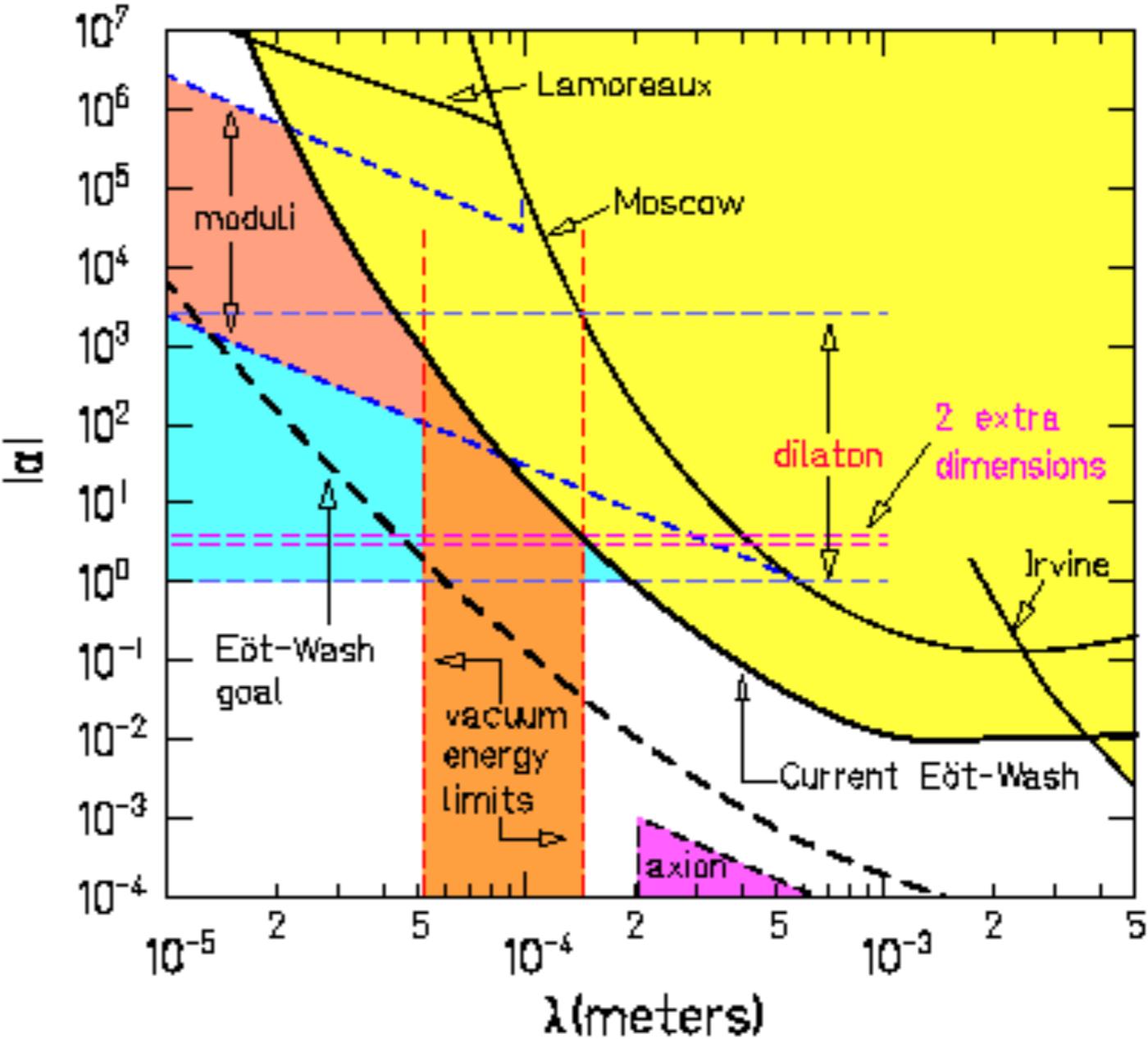
First term gives Newton's inverse
square law, second term is deviation
from standard gravity

tightest constraint comes from
satellite experiments

$$\beta^2 \leq 4 \cdot 10^{-5}$$



the scalar field must be screened to agree with observations



Chameleons

Chameleon field: field with a matter dependent mass

A way to reconcile gravity tests and cosmology

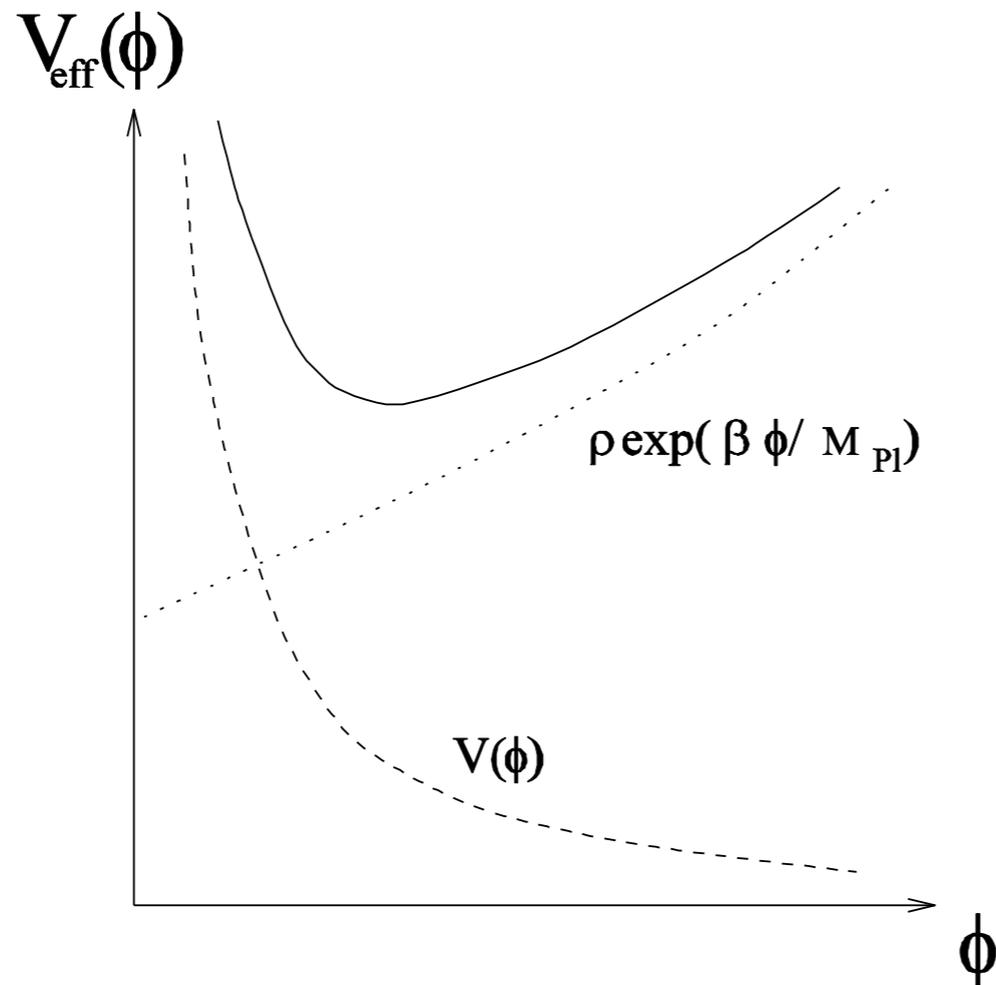
nearly massless on
cosmological scales

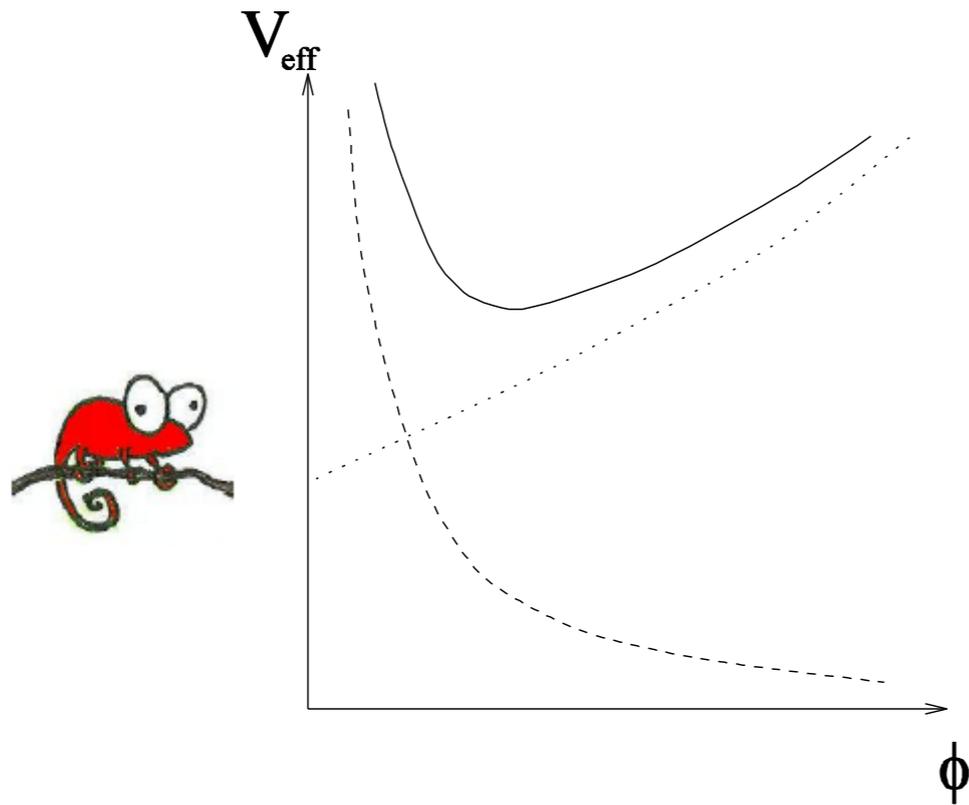


massive in the
laboratory and
solar system

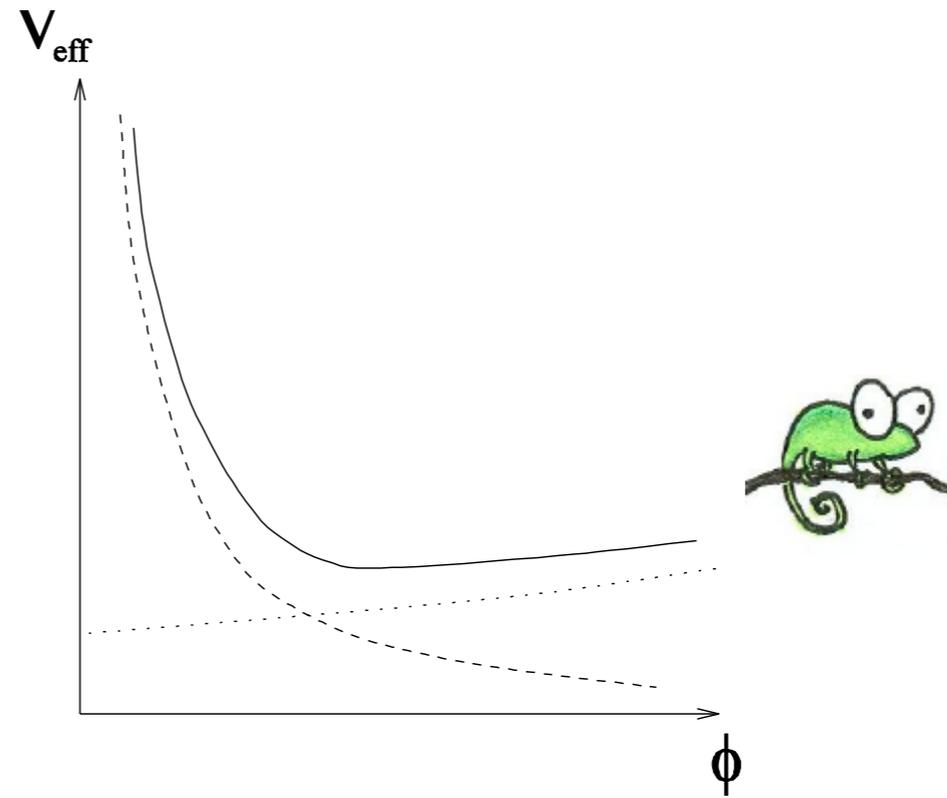
you can have the cake and eat it!

There is an environmental effect: when coupled to matter the potential depends on the ambient matter density as well





Large ρ



Small ρ

mass is proportional to the second derivative of minimum of the potential
Hence it can be heavy when ρ is large and light when ρ is small

$$m_{\phi}^2(\rho) = \partial^2 V(\rho) / \partial \phi^2$$



The range of the force depends on the inverse of the mass

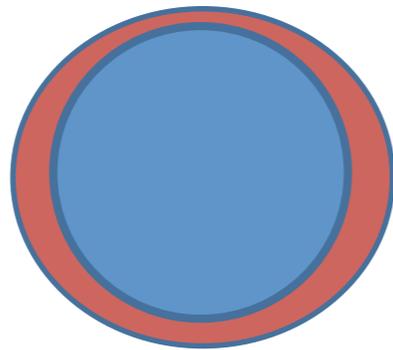
One can throw the tennis ball much further than the medicine ball

Imagine throwing the tennis ball in mud or treacle! It becomes much more difficult

The range of the chameleonic force depends on the environment!

When objects are big enough/dense enough, or if they are surrounded by big objects, the field is screened. Inside it is nearly constant apart from inside thin shell whose size is inversely proportional to Newton's potential at the surface

Thin shell



the fifth force is proportional to the size of the thin shell

$$F_{\phi} \propto \Delta R / R \Phi_N$$

because it depends on the gradient of the field

what is dense enough for the chameleon effect?

The environment dependent mass is enough to hide the fifth-force in dense media such as the atmosphere, hence no effect on Galileo's Pisa tower experiment

$$\rho \approx 10^{-4} \text{g/cm}^3$$

It is not enough to explain why see no deviation from Newton's gravity in the lunar ranging experiment

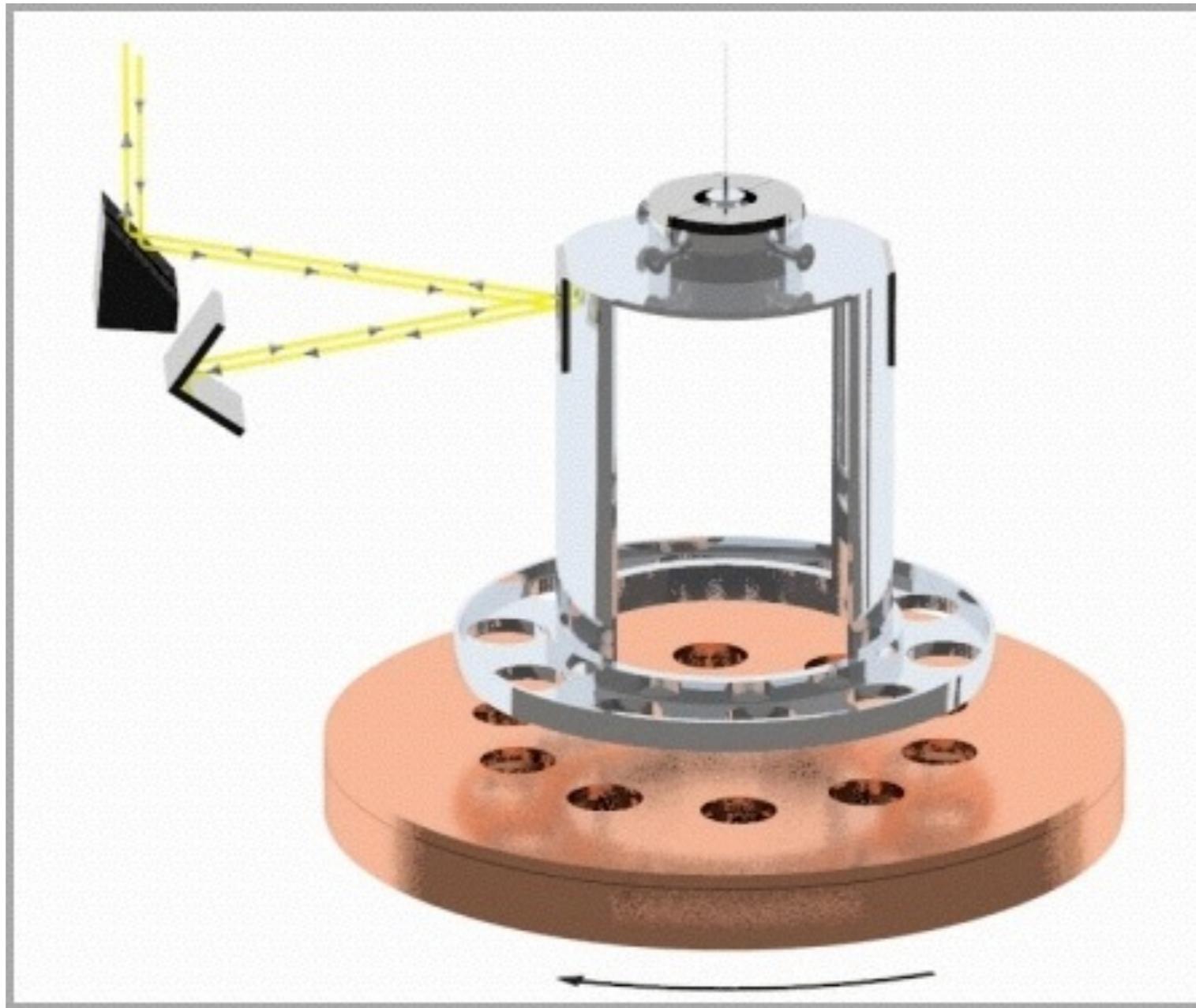
$$\rho \approx 10^{-22} \text{g/cm}^3$$

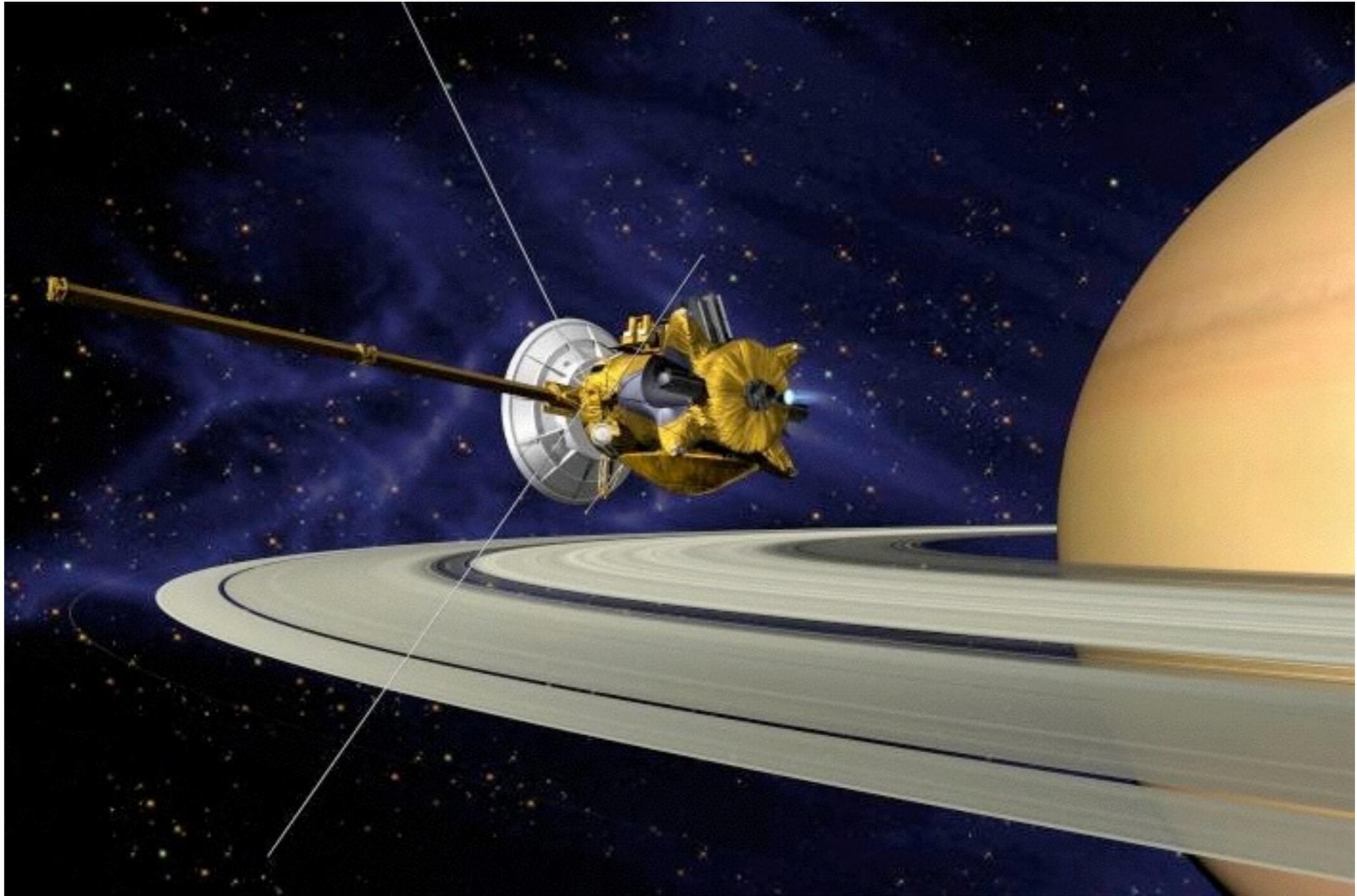
It is not enough to explain no deviation in laboratory tests of gravity carried out in the 'vacuum'

$$\rho \approx 10^{-14} \text{g/cm}^3$$

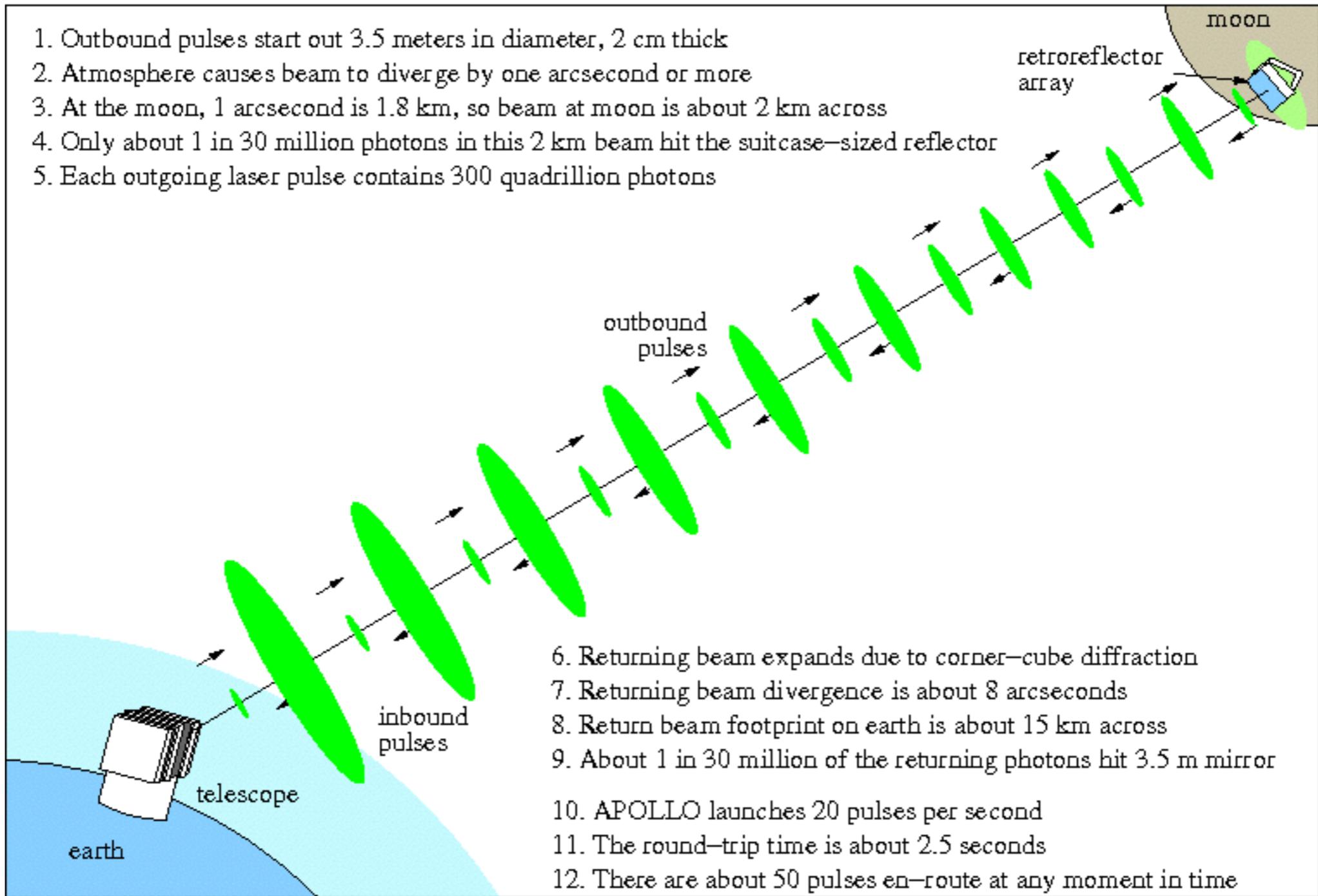
We can use this to put constraints on the theory

Eot-Wash Experiment









Black Holes

These are solutions of Einstein Equations of General Relativity.

They can be described by the Schwarzschild metric

$$ds^2 = -c^2(1 - R_s/r)dt^2 + (1 - R_s/r)^{-1}dx^2 + r^2d\Omega^2$$

where $R_s = 2M_{BH}G/c^2$

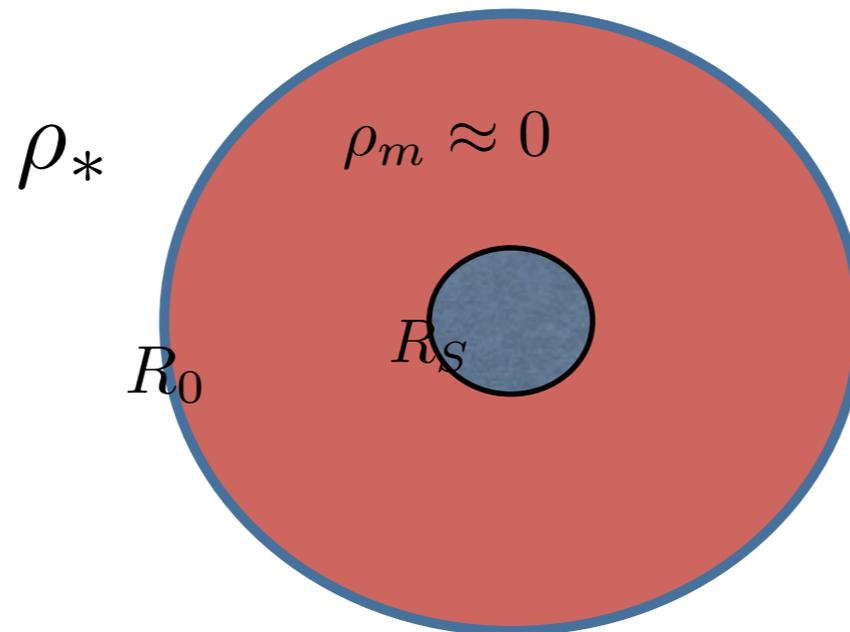
is the Schwarzschild radius, M is the mass of the black hole, G is Newton's constant and c is the speed of light

Note the singularity at $r = R_s$

A particle orbiting a black hole with radius $r < 3R_s$ is unstable

There are black holes in the centre of most galaxies. For example in the centre of our galaxy there is a black hole of 10^6 solar masses.

Particles whose orbits are less than the smallest stable orbit, or innermost stable orbit, are sucked into the black hole, leaving approximate vacuum surrounding the black hole. Beyond this there is the accretion disk of matter.



Screened ‘Hair’ on a Black Hole

Consider the effect of screened modified gravity in the presence of a black hole. There is a ‘no scalar hair’ theorem for black holes, but this is for idealised conditions where there is no matter and the scalar field is constant.

This is not the case in screened modified gravity.

with Rahul Jha, Jessie Muir and Ruth Gregory

The set up

We consider an idealised situation with a Schwarzschild metric and black hole horizon at

$$R_s = 2M_{BH}G/c^2$$

In the absence of the scalar field the region I is vacuum

$$R_s < r < R_0$$

with a constant matter density ρ_* in region II

$$r > R_0$$

This mimics a black hole with an accretion disk

We solve the field equations in regions I and II, match at the boundary for the field and its derivative. We have done this for chameleons.

$$\square\phi = \frac{1}{R^2} \frac{d}{dR} \left[R^2 \left(1 - \frac{R_s}{R} \right) \frac{d\phi}{dR} \right] = \frac{\partial V_{\text{eff}}(\phi, \rho)}{\partial \phi}$$

Taking chameleon potential and coupling as

$$V(\phi) = V_0 \phi^{-n} \quad A(\phi) = e^{\beta\phi/M_p}$$

and
$$V_{\text{eff}} = V(\phi) + \rho A(\phi)$$

Region I Taking
$$\phi(r) = \phi_h + \delta\phi(r)$$

We get
$$\frac{d\phi}{dr} \approx \frac{V_{,\phi}(\phi_h)}{3} \left(\frac{R_s^2}{r} + R_s + r \right)$$

$$\phi(r) - \phi_h \approx \frac{V_{,\phi}(\phi_h)}{6} \left(2R_s^2 \ln \frac{r}{R_s} + (r - R_s)(r + 3R_s) \right)$$

In the thin shell limit the field will vary rapidly in the region

$$R_0 < r < R_1 \quad \Delta R = R_1 - R_0 \ll R_0$$

and $\phi(r) = \phi_*$ for $r > R_1$

$$\frac{1}{2} \left(\frac{d\phi}{dr} \right)^2 = V(\phi(r)) - V(\phi_*)$$

Matching we finally get

$$\phi_h \approx \left[\frac{nV_0 R_0^2}{6} \right]^{\frac{1}{n+2}}$$

the value of the scalar field
at the black hole horizon

The black hole scalar charge is given by

$$Q_\phi = \lim_{r \rightarrow R_s} -r^2 \frac{d\phi}{dr}$$

Inserting our results this gives

$$Q_\phi = -nV_0 R_s^3 \phi_h^{-(n+1)} = -nV_0 R_s^3 \left[\frac{6}{nV_0 R_0^2} \right]^{\frac{n+1}{n+2}}$$

Note that $Q \rightarrow 0$ as $R_0 \rightarrow \infty$

giving back the no hair theorem

What Next?

We can compute the extra, scalar force

$$\frac{F_\phi}{F_N} = \frac{\beta R_S^2 V_0^{1/3}}{R_0^{4/3} M_{Pl}}$$

For realistic black holes, infalling matter should feel

$$F_\phi + F_G$$

The scalar force is tiny for the black hole in our galaxy, but could be significant for supermassive black holes with mass

$$M_{BH} \approx 10^{9-10} \text{ solar masses}$$

Outlook

There are many ways we can test modified gravity theories. Just because the fifth force is screened in the solar system doesn't mean you can't observe the effects cosmologically.

We have looked for deviations in the predictions for large scale structure and the CMB. These could be detected with future surveys. We are also looking at predictions for gravitational waves.

Our results have tested General Relativity in regimes it would not have been tested in previously. Who knows there may be deviations in future!