Winter school on closed geodesics

Neuchâtel, February 6–14, 2009

This workshop addresses Master and PhD students interested in geometry or dynamics. The goal is to understand some principal theorems on the existence and the number of closed geodesics on a compact Riemannian manifold.

The search for closed geodesics is an old and beautiful topic in Riemannian geometry, with many applications to Riemannian geometry in the large. Geodesic flows are special Hamiltonian flows, and finding closed geodesics is finding closed orbits of Hamiltonian systems in a model case. In fact, closed characteristics of a classical Hamiltonian system of the form "kinetic energy + potential energy" can be viewed as closed geodesics of a Riemannian metric, the so-called Jacobi metric. The methods used in finding closed geodesics (variational methods, Morse and Lusternik-Schnirelman theory, Hamiltonian dynamics, hyperbolic dynamics) are profound and useful for many other problems.

We plan to cover the following topics.

- In every nontrivial free homotopy class there exists a closed geodesic.
- On every closed Riemannian manifold there exists at least one closed geodesic.
- For every Riemannian metric on S^2 there are at least 3 embedded closed geodesics.
- Growth of the number of closed geodesics below a given length (for both manifolds with finite and infinite fundamental group).
- The classification of Wiedersehen-Mannigfaltigkeiten (manifolds all of whose geodesics are closed).

If time permits, we will also briefly address the following topics:

- The geodesic flow of a Riemannian manifold of negative curvature is ergodic.
- The relation between the growth rate of closed geodesics and topological entropy.
- The relation between the length spectrum and the Laplace spectrum.

Prerequisites: It would be good if each participant has a sound knowledge on basic topics in Riemannian geometry, such as manifolds, Riemannian metrics, Levi-Cività connection, exponential mapping, parallel transport, the definition of geodesic and what they are in $\mathbb{R}^n$ and on the round sphere, the Hopf-Rinow theorem, the definition of the geodesic flow, curvature (sectional and Ricci); as presented, for instance, in Chapters 2 and 3 of

Gallot, Sylvestre; Hulin, Dominique; Lafontaine, Jacques. *Riemannian geometry.* Third edition. Universitext. Springer-Verlag, Berlin, 2004.

We also assume that the participants know singular homology. Knowing finite dimensional Morse theory (until the Morse inequalities, as exposed in Milnor's classical book) would be useful, but is not indispensable.

Speakers: The main part of the course will be given by Urs Frauenfelder (LMU München), Peter Albers (ETH Zürich) and Felix Schlenk (Uni Neuchâtel). Individual lectures will be given by Oliver Fabert (LMU München), Muriel Heistercamp (Uni Neuchâtel), Fabien Ngo (ULB Bruxelles), Gregor Noetzel (Uni Leipzig), Will Merry (Cambridge University), Eugene Volkov (LMU München), and Jan Wehrheim (LMU München). We plan to have an exercise section each day, and an excursion to the Creux-du-Van.

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