Balkan Mathematical Olympiad 2020 – A Virtual Event

Saturday 31st October – Thursday 5th November 2020

Organised by **Romania** (and theoretically held in Buşteni)

A Report by Robin Bhattacharyya (United Kingdom Team Leader)

The **UK** has attended and competed in the **Balkan Mathematical Olympiad** every year since **first appearing in 2005** (when the competition also happened to be held in Romania).
This Olympiad is an annual competition for 11 member countries from southeast Europe and for a few other invited guest teams. The host nation (which is always a member country) changes each year, and in 2020 it was Romania's turn to host again.

Like the International Mathematical Olympiad (the IMO), this is a competition to inspire, challenge and develop very bright students. The focus is rather different from normal school maths, being about problem solving, and the strategy needed to solve the questions is not at all obvious.
The subject focus is in four broad areas: algebra, combinatorics, geometry and number theory.
Just as in the IMO, each country selects a **team of 6 school age students**. But unlike the IMO, there is only **one exam** at the Balkan Olympiad; this exam lasts for four and a half hours and it is made up of **4 questions**, each one to be marked out of 10 points.

The UK makes a particular effort to use this competition to give experience to students who might be in contention for places in other competitions in the future but haven't had much experience yet.
(Indeed, more than two thirds of all the students at this year's Balkan Olympiad had competed at the (virtual) 2020 IMO a few weeks earlier, but none of the UK's Balkan team had been involved in that.)

The rules of the Balkan Olympiad state that each year the contest should be held between April 25 and May 10. **Romania had agreed to host the contest in the spring of 2020, and the venue was to be Buşteni, in the (Carpathian) mountains** but, because of the pandemic, that didn't work out.

What follows is a timeline of what did ensue, in this strangest of years.

Early March 2020 – the Romanian Master of Mathematics competition finishes and the UK team returns home, just as normal. The UK is planning to send a team to the Balkan Olympiad, as usual.

Saturday 14th March – the UK has by now **decided to pull out** of the Balkan Olympiad, unsurprisingly I have to say. Day by day, in this past week, coronavirus concerns have grown and grown in the UK, and this decision, which didn't at all feel inevitable just a week ago, now does seem so, regrettably.

(And indeed, just over a week later, the UK was to go into a full 'lockdown' because of the disease.)

The European Girls' Maths Olympiad (EGMO) quickly decides to become an online event, but the Balkan Olympiad **is postponed** instead, perhaps with the hope that a physical event could happen in the autumn in Romania.
There's much else to concern ourselves with at this point, so I don't give much more thought to the Balkan Olympiad, for the moment.

The IMO is also postponed until the autumn; in its time slot in the summer a speedily arranged 'Cyberspace Mathematical Competition' takes place online, for teams from 75 countries.

<u>Six months after pulling out…</u>

Wednesday 16th September – I am asked whether I'd be interested in leading the UK team in the now **online Balkan Mathematical Olympiad**, to be held in six and a half weeks' time.
(I hadn't actually known that the competition was going to happen, or that the UK would enter.)

Thursday 17th September – I get permission from my school for 2 days away from work at the start of November, so that I would be able to mark the UK team's work and then communicate with the Romanian coordinators to decide the final marks for the team.
Coronavirus related potential absences mean that my school is naturally keen to have as many staff as possible physically in school, and I am not permitted more than two days off.

Sunday 20th September – I am confirmed as the Leader of the UK team.
(This will be the fourth time for me helping at a Balkan Olympiad (my third time as Leader).)

Sunday 27th September – The UK will ask whether its team might be allowed not to have to meet up in person for the exam, i.e. so that they wouldn't have to all do it in the same location – the rules of the competition actually say that each team should be together at a physical exam centre on the exam day of November 1.
James Gazet is kindly communicating with the Romanian organisers about this matter on behalf of the British Maths Olympiad organisation.

Thursday 1st October – There was a training camp for about two dozen UK students back in April, on Zoom (normally it's the 'Trinity Camp' which takes place physically in Cambridge each Spring, for nearly a week). On the basis of scores in the tests after this camp, the **UK team is now selected** (principally by Kasia Warburton) from the list of the still eligible students. The team members will be:

Wilf Ashworth	**Sutton Grammar School**
Ojas Gulati	**Tiffin School**
Sean Jaffe	**Winchester College**
Linus Luu	**St Olave's Grammar School**
Eleanor MacGillivray	**King's Ely**
Jenni Voon	**Landau Forte College, Derby**

Ojas is, even now, only in Year 11, while Eleanor is in Year 12 (and the others are all in Year 13).

Wilf has been to Romania for a competition already in 2020 – the 'Romanian Masters of Maths' competition – and both he and Jenni represented the UK in the online 'Cyberspace Mathematical Competition' in the summer (which Kasia and I also assisted with).

Monday 5th October – Kasia confirms that she is able to be the UK Deputy Leader; this is the third (or fourth) competition for her in this role in 2020!!

So we have:

Robin Bhattacharyya	**Loughborough Grammar School**	**Leader**
Kasia Warburton	**Trinity College, Cambridge**	**Deputy Leader**

Tuesday 6ᵗʰ October – James Gazet says that we do indeed have **permission for the UK students not to take the exam physically together**; we just need to vouch that the students are doing it securely. This should make things easier for us – trying to find a suitable venue at short notice, with the virus very much on the increase right now, seems best avoided.

Friday 9ᵗʰ October – All 6 selected students have agreed to participate in the Olympiad, being free on Sunday 1ˢᵗ November for the exam (and also available for some Zoom meetings on some days beforehand for information/training).
So we definitely have our team now, with three weeks to go.

Sunday 11th October – I send the team quite a few past Balkan Olympiad questions, that the UK IMO Leader, Dominic Yeo, has given me to help with their training. Getting practice in all four of the main topic areas is important, as all of these will be represented in the exam.

Wednesday 14ᵗʰ October – all the students have responded, sending in photos of themselves for use in the virtual Opening Ceremony, which will be happening on the day before the exam.

I find out that we're past the original deadline for officially registering the team, so this really needs to be done now, student by student, on the official website for this year's Balkan Olympiad. I register us on **Saturday 17ᵗʰ October**, having first checked that our students give their permission for their photos to appear – and within a couple of days we all appear on the official website.

Sunday 18ᵗʰ October – For the Opening Ceremony, I send in a 'slide' showing photos of the UK team and five views of the UK. I choose, partly to reflect where we're all living, pictures of London, Cambridge, Ely Cathedral, the Headstone Viaduct at Monsal Head (in Derbyshire) and the Helvellyn area (in the Lake District). The Romanians remind me that the names of our students must also be on the slide, so I edit and send it again.

Tuesday 20ᵗʰ October – I receive an invoice for the Olympiad's (modest) guest country entry fee.

Thursday 22ⁿᵈ October – I have been working my own way through the practice questions from previous Balkan Olympiads; this has taken a while.
Finding particularly neat solutions to some of the geometry has been a bit of a challenge.
Now I send around the hints to these questions (which were again sent to me by Dominic Yeo).
The team have also been busy with these, Eleanor for example recently e-mailing in about the final step in her approach to one of the geometry questions.

Sunday 25ᵗʰ October – We have the first team Zoom meeting this afternoon, a week before the exam. The meeting with the team lasts for 40 minutes (the Zoom maximum for my free account). Inequalities are often seen in the Balkan Olympiad and I mention Schur's inequality, which some of the students have also researched; we also look at some geometry – the team have been making some very good progress there. I experiment with my new graphics tablet, to illustrate.

Thursday 29ᵗʰ October – I find out that the UK team definitely has to be **on camera while taking the exam** on Sunday.
Therefore, I sign up to Zoom Pro for a month, to allow for meetings of much longer than 40 minutes.

Friday 30th October – There is an official 'camera test' this morning with the Romanian organisers, from 9am. I do this with a mini iPad filming a (test) Zoom meeting on my computer screen.
It's actually my first ever Google Meet meeting; indeed, I had never done any videoconferencing of any sort before March of this year. I have to put the address of our 'exam centre' onto the chat for the Google Meet meeting (the exam centre is in reality my children's bedroom!).

I send the team the files for their official badges (to be shown on camera just before the start of the exam) and for the **official paper** for neat solutions to be written on, the official paper for rough work, the sheets for any queries and the official cover sheets for each student for each question (to be the first page of each pdf file submitted); the students have to print this all out, including probably at least 20 neat pages and 20 rough pages of paper – I hope that's going to be OK for them.

Saturday 31st October – We seem all set now for tomorrow's exam. The final parental permission comes in for the students to be on screen as they do the exam.

The **Opening Ceremony** is on the official Balkan Olympiad 2020 website by 1pm (it's recorded). There are some fine views of Romania, not least the Buşteni area, some words about maths being behind the technology both to make this event possible in these times of limited travel and also to keep the world economy going, some words from Preda Mihăilescu, who solved the famous 'Catalan's Conjecture' problem nearly twenty years ago, and then, at the end, a 'parade of the nations' showing the teams, many in photographs/video – including a few seconds for the UK team.

Our team have a Zoom meeting in the afternoon, for final reminders about the exam and being filmed, using the correct sheets of paper, and making the pdf files afterwards. Everyone seems well.

Sunday 1st November 2020 – **The Exam Day**. The question paper itself is sent around to all the team leaders at 6am, for translation – which is not necessary for us, as English is the official language of this competition. I get up at about 7am and I check the exam paper – no queries on the wording seem likely from the UK team. I send the exam to Kasia. And I try to wake up. Normally all the team leaders are involved in the selection of the exam questions, so they would know before now which questions had been picked; but in the current circumstances some things are operating differently.

There's some e-mail discussion among organisers and leaders about the wording of Q4: essentially 'smallest integer…that has more positive divisors than a_n' becomes in the new revised version 'smallest integer…that has more positive divisors than a_n has', and this version is released just before the exam starts.

I start a long meeting on Zoom with Kasia and the UK team from 8.20am. And I join a Google Meet meeting with the Romanian organisers and the other team leaders / their exam room webcams, from 8.30am. I am slightly relieved to see all of our students on Zoom by about 8.40am, as instructed. Their official badges are held up to their cameras at about 8.45am to prove that they are who we say they are. Everyone seems ready. I see some other teams (through Google Meet) who are spread out in their physical classrooms, with many of the students wearing masks.

I e-mail the exam paper to the UK students at 8.59am.

The 2020 Balkan Mathematical Olympiad contest paper:

Problem 1. Let ABC be an acute triangle with $AB = AC$, let D be the midpoint of the side AC, and let γ be the circumcircle of the triangle ABD. The tangent of γ at A crosses the line BC at E. Let O be the circumcentre of the triangle ABE. Prove that the midpoint of the segment AO lies on γ.

Problem 2. Denote $\mathbb{Z}_{>0} = \{1, 2, 3, \dots\}$ the set of all positive integers.
Determine all functions $f : \mathbb{Z}_{>0} \to \mathbb{Z}_{>0}$ such that, for each positive integer n,

    i) $\sum_{k=1}^{n} f(k)$ is a perfect square, and
    ii) $f(n)$ divides n^3

Problem 3. Let k be a positive integer. Determine the least integer n, with $n \geq k + 1$, for which the game below can be played indefinitely:

    Consider n boxes, labelled $b_1, b_2, \dots, b_n$. For each index i, box b_i contains initially exactly i coins.
    At each step, the following three substeps are performed in order:
    (1) Choose $k + 1$ boxes;
    (2) Of these $k + 1$ boxes, choose k and remove at least half of the coins from each, and add to the remaining box, if labelled b_i, a number of i coins.
    (3) If one of the boxes is left empty, the game ends; otherwise, go to the next step.

Problem 4. Let $a_1 = 2$ and, for every positive integer n, let a_{n+1} be the smallest integer strictly greater than a_n that has more positive divisors than a_n has. Prove that $2a_{n+1} = 3a_n$ only for finitely many indices n.

$$Time: 4\,{}^1\!/_2 \ hours$$
$$Each\ problem\ is\ worth\ 10\ marks$$

I have the iPad on Google Meet, filming (and filmed by) a PC which has the UK team showing on its screen on our Zoom meeting. I frequently peer over to look at the screen to see if anyone in the UK team is trying to attract my attention. After half an hour, when I tell them that they can no longer put in queries about the questions (not that anyone has done so), I mute myself permanently.

The exam seems uneventful. **Each time I look at the screen, I see people very deep in thought**. Quite near the end, a couple of students have iPad battery issues: Jenni disappears for a moment to find a charger, and Linus shuts down his machine and is off the meeting for a moment; I wonder whether he has finished, but then a minute later he rejoins – he had moved to another room, because his battery was going too.
The exam finishes at 1.30pm, when I tell the students to stop working.

From 1.30pm until about 2.30pm the students' next challenge is to make and send pdf files of their work. They need one file for each question, starting with an official cover sheet listing how many other pages they have, followed by their neat work for that question and then finally their rough work. Then each team member will **e-mail their files to me**. They're still on camera while they do this (and we talk through the process a bit). Some big files take a while to send, but eventually we

get everything, when Jenni's last file arrives. I check what has been sent, as it comes in, on my e-mail, on a second iPad. That's lots of use of technology here (at least it seems so to me)!

From about 2.30pm to 3.30pm, I check the 24 files, page by page, making sure that all is there and in the correct order; much has been written. Indeed, Sean ran out of the printed official paper and had to use a couple of sheets of random paper too, but that's not surprising given the amount of work everyone has been doing! I rename files where necessary, and upload them onto the Romanian url (while still on camera myself on this *very* long Google Meet meeting). It's a relief when that's all done successfully, and the head of the Olympiad's IT, **Silviu Vasile**, puts in the chat that he has all the UK files**. I can finally leave the Google Meet meeting after about 7 hours!**

Then there's a bit more to do as I read e-mails from four of the team, saying in summary how they think they've done, and I e-mail Kasia this information (plus what I've found out about how the other two on our team seem to have done, from looking at their scripts). I send Kasia a few scripts she wasn't sent by the team, and there are a couple of other things to do. By 4.30pm, Kasia and I have decided that she will in charge of marking Q1 and Q4 and I'll be taking the lead with Q2 and Q3.

I finally have my lunch at 4.45pm!! This after eight and a half hours on machines, out of the way in my children's room (Matthew and Edward can use it again now!). It's nice to see my family again.

Everyone in the UK team has had some success in the exam, which is good news. They all seem to have done Q1 (geometry), but we'll have to check; they also all seem to have solved Q2 (algebra). On Q3 (combinatorics) there is very promising looking work from Wilf and Jenni. Linus has lots of good work on Q4 (number theory), but has he finished it off? Wilf and Sean also really engaged with Q4, but didn't quite get to the heart of the question; we'll have to see what their work will be worth.

The Romania B team leader e-mails me at 10.30pm with his team's scripts for Q1 – ahh!!
I'd found out earlier in the day that Question 1 was a UK submission, which I thought was nice; I had completely forgotten about who had to mark the scripts for the host team(s) for this question!

In fact, Q1 was set by the UK (Sam Bealing), Q2 by Albania, Q3 by Cyprus and Q4 by North Macedonia.

Monday 2nd November – This morning I read the first 5 Romania B geometry (Question 1) solutions, which I was too tired to do last night; I have some time because I don't have the Romania A scripts yet (which need to be coordinated first) – I e-mail about this. By 11am we have all the official mark schemes to look at. I receive the Romania A geometry questions at just after 12.30pm, on a Google drive – they had tried to send by e-mail previously but something went wrong, quite likely at my end, with the files being large.

3.30pm: Kasia and I **coordinate Romania A on Question 1**; we work with the Romanian coordinators and the Romania A leaders to decide the marks. I want a translation from Romanian into English of the last paragraph of one script (ROU5), I am confused about some trigonometry in one (ROU4) then work out that part of a formula has split over onto a second physical line, and I note that angles named x and y are mistakenly swapped halfway through by another student (ROU6); but it doesn't matter: everyone has a proper solution and it's going to be 10 marks each for the Romania A team.

5pm is the official time for **coordination of the UK scripts for Question 2** (with a half hour long session scheduled, the same as for all the other questions) but we actually do most of this from about 4.15pm, with Kasia and I speaking with coordinator (and IMO Gold medallist) Mădălina Persu – who like Kasia and me had been a coordinator at the IMO in Bath (and South Wales) in 2019. (Mădălina had e-mailed us, asking if we could start this meeting early.) I explain a couple of minor points, in a couple of scripts. Then Kasia realises that Linus's error is just having the 'close brackets' and 'squared' symbols swapped around and also having another 'squared' symbol missing. So, all is fine actually: it's **10 marks for all our students on this question**. I can let our team know the good news!

In the evening, I get to read more of the UK scripts (now that we're finished deciphering the Romanian language geometry scripts).

Tuesday 3rd November – We are pretty busy with coordination meetings this morning; and Kasia and I are e-mailing each other our final thoughts right up until a minute before each session starts. (Also, Kasia has some trouble joining the coordination Google Meet meetings – I should have just e-mailed her all the links earlier, but I do so now.)

UK Question 1 coordination is at 10am. Kasia takes the lead on this question. She explains a queried step in Sean's long solution (which has some irrelevant bits in it, but works), and it's all fine; Eleanor has a missing angle chase so **she gets 7/10, and everyone else scores 10/10**. A variety of approaches has been used.

UK Question 3 coordination is at 11am. A fast one – the coordinators are very happy with Wilf's solution, which they say is 'crystal clear', and we note that Jenni hasn't explicitly given an example of how in the optimal case the game can actually go on forever; but her inequalities force what such an example could possibly be, so she gets some credit for that part of the question too, and her mark is 8/10 overall. There are no marks for any of the other students: 'Sorry…sorry,' says the coordinator, very politely! Ojas and Linus had written a few thoughts down, but not enough of what is on the mark scheme for either of them to get any credit.

UK Question 4 coordination is at 12pm. Linus hasn't finished this question off, but he is in fact most of the way there, using inequalities for indices, which he finds using the primes 5 and 7; he notes that for large powers of 5 and 7 these inequalities are inconsistent, so the power of 5 dividing a 'special' a_n must be absolutely bounded above; then he shows that the powers of 2 and 3 are consequently bounded too – but he hasn't shown that the number of distinct primes dividing an a_n in such a situation is also bounded (he has also omitted to actually prove one of his (correct) early statements). His is not quite the standard approach, but it will work. He gets 6/10.

Wilf has inequalities from looking at multiplying a_n by 5/4 and 9/8 and 27/25 and he states that for sufficiently large powers of 2, 3 and 5 dividing a_n there will be a contradiction; it's a useful approach but the second of his three inequalities is essentially the wrong way around, so it's not quite there. He gets a mark of 3/10. Sean has done a lot of work on this question. In rough, he has inequalities from multiplying a_n by the two fractions that appear in the standard solution, but unfortunately he doesn't successfully combine them together on paper or notice that doing this would give a contradiction for big powers of 2 and 3. He gets a mark of 2/10. There are no marks for our other students on this question; Eleanor has (appropriately enough for contestant UNK5 on Question 4) multiplied by 5/4, and she has derived an inequality; Ojas also engaged sensibly with this question.

The **Romania B team's geometry is coordinated** at 12.30pm. We all quickly agree that all the students have solved it, even in the quite long inversion proof given by ROUB6. So, it's 10 marks for each of these students.

Then we're done! It's been quite busy and tiring during the last few days, but there isn't too much else to do now.

I let the UK students know all their marks. But we don't yet know how any other team has done.

Later on, in the evening, there's a Zoom meeting for the UK team to reflect on the whole experience and to look at where marks have been won and lost; everyone except Wilf and Ojas is able to join this quickly arranged meeting.

Here are some of the thoughts which were expressed in the meeting:

Linus thinks that doing the exam on camera made it an **'intense experience'**.
Sean says that having an exam early in the morning on a Sunday during half-term was not an ideal time for him – he normally gets up rather later in the holidays. But when Kasia mentions that IMO 2020 (in September) had all the teams overlapping in time for at least a little bit of each exam, so that New Zealand had to finish at about midnight, Eleanor points out that she might have to get up at midday on every day for about a month beforehand, to make finishing that late work for her!
Linus thinks that the end of the half-term holiday is not ideal for him for this competition, and it is indeed a busy time of year, with the TMUA exam to be sat on Thursday, and the MAT tomorrow (for Sean); and our own Senior Maths Challenge will be happening from Monday to Thursday of this week. But at least there's no direct clash between any of this and the Balkan Olympiad.

Someone points out that not getting a full question correct for quite a while from the start of the exam is a bit of an awkward feeling; this is actually one of the reasons why we want to use the Balkan Olympiad – to give people the exam experience and practice that only a real competition can provide, to help them in the future. But things do indeed feel better after solving a full question.
The consensus in the UK team is that Question 2 was the easiest of the questions this year. But I had wondered if anyone didn't already know the sum of the first n cubes formula, and Eleanor (who is not yet in Year 13) confirms that she had to conjecture and prove this formula during the exam.

Jenni says that she knew she hadn't produced the 'construction' in Q3, but she'd just run out of time for this at the end of the exam.
Linus was surprised that his list of deductions in Q4 was worth as many as 6 marks; but he shouldn't have been surprised – he was close to a full solution.
And **his advice to future Balkan Olympiad students is for them to do lots of practice questions**.
Kasia mentions that Sam Bealing (the author of Question 1 for this exam) and she coincidentally live literally just up- and down-stairs from each other (in Trinity College, Cambridge)!
We finish the meeting after about half an hour. (Kasia has been missing 'Bake Off' for this)

Here are all **the UK results**, which were agreed by about 12.15pm earlier today (Tuesday 3rd):

Wilf Ashworth	**33**:	10, 10, 10, 3
Ojas Gulati	**20**:	10, 10, 0, 0
Sean Jaffe	**22**:	10, 10, 0, 2
Linus Luu	**26**:	10, 10, 0, 6
Eleanor MacGillivray	**17**:	7, 10, 0, 0
Jenni Voon	**28**:	10, 10, 8, 0

Wednesday 4th November: I'm back at my regular job now, teaching at Loughborough Grammar School, which means getting up at about 6.30am and then driving into work. I have, like everyone else, been working from home for the duration of this Olympiad.

The results of all the students, from all the countries, are e-mailed to all the leaders for final checks, in the evening. Medal boundaries have been decided too: **32 points for Gold**, **25 for Silver**, **and 20 for Bronze**. This means that the UK has **1 Gold**, **2 Silvers**, **2 Bronzes** and **1 Honourable Mention**. It's a fine haul, and one of our best ever at the Balkan Olympiad:

Wilf Ashworth	**33**	**Gold**
Ojas Gulati	**20**	**Bronze**
Sean Jaffe	**22**	**Bronze**
Linus Luu	**26**	**Silver**
Eleanor MacGillivray	**17**	**Honourable Mention**
Jenni Voon	**28**	**Silver**

We come 7th out of the 18 teams on total team score, which is the **UK's equal second best performance at any Balkan Maths Olympiad**.

This has been the first international contest for Ojas (who is in Year 11), Eleanor (in Year 12) and also Sean and Linus.

(Reminder: The UK deliberately picks a team for this competition comprising of students who have little or no international Olympiad experience - this is partly to give people that experience, for the future, and partly to give more people the chance to compete. In particular, we will not send someone to a Balkan Olympiad if they have been to a previous Balkan Olympiad; and our team here has none of the UK IMO team members from September 2020 (or indeed any of the reserves) – while 14 of the other 17 teams here have included at least 4 members of their IMO team (of 6).)

Thursday 5th November: The **second national lockdown** begins in the UK today, but schools and universities remain open this time.
To be honest, I don't notice too much difference in the amount of traffic, when driving into work.

The full competition results are all up on the official Balkan Olympiad 2020 website by 12pm.
And the **Closing Ceremony** (featuring the medal winners) is put onto the website just before 4pm.

Wilf has won only the third ever UK Gold at a Balkan Maths Olympiad; and he's in equal 4th position overall in this year's competition (jointly with four other students).

The top scores in the competition are in fact: 40 and 40 – from Edis Memiş of Romania and Dobrica Jovanović of Serbia, who were also both Gold medallists from this year's IMO – then 34, 33, 33, 33, 33 and 33 (including Wilf).   [Incidentally, our physical medals arrive in the UK in mid-December.]

Jenni is the second ever UK girl to get a Silver at the Balkan Maths Olympiad.

The two girls in our team are in fact in **first and fourth place among all the girls in this year's Balkan Olympiad**, which is very pleasing. <u>**Yuka was the highest placed girl in the whole IMO (2020), and now Jenni is the highest placed girl in the whole Balkan Olympiad (2020)**</u>.

Here are the **team results**:

(1) Serbia	177
(2) Romania	176
(3=) Bulgaria	170
(3=) Turkey	170
(5) Italy	155
(6) Romania B	148
(7) UK	**146**
(8) Moldova	140
(9) Azerbaijan	139
(10) Kazakhstan	134
(11) Saudi Arabia	128
(12) Greece	127
(13) Bosnia	118
(14) N. Macedonia	113
(15) Turkmenistan	87
(16) Cyprus	67
(17=) Albania	55
(17=) Montenegro	55

The leaders have been sent the (short)list of questions which were submitted for use in this competition; it has 18 questions on it, of which 5 were submitted by the UK.
(There were also 4 other questions submitted, which weren't released to leaders on this (short)list because the proposers wanted to give themselves the chance/option to try to use them in another competition at some point in the future.)
The list is confidential until the next Balkan Maths Olympiad, whenever that will be – there is still no decision on who might host in 2021, and exactly when.

Some very **brief comments on the four problems** chosen for this year's contest:

Q1 – (Geometry) There were many ways to solve this question, some fairly short and some longer. Usually, solutions proved and then used a fact about where C is in relation to B and E (an accurate diagram can help with this). Some solutions chased angles and found similar triangles, while others also made use of the sine rule. The UK team used many different methods (as did the Romanians).

Q2 – (Algebra) Some trying with small numbers seems to suggest that the choice can be very restricted at each step. This problem can fall out pretty quickly if you spot the right thing; trying enough algebraic reformulations can help one to stumble onto a method which works.

Q3 – (Combinatorics) It may well take a while here to get any intuition about what sort of function will turn out to be the correct one – extensive small number trials can be very useful in formulating your guess, and for understanding what an optimal process could be.

Q4 – (Number theory) Where to start?? Perhaps multiplying a_n by 3/2 (from the question) suggests to look at powers of 2 and 3, and which particular multiple(s) of a_n could be 'better' than a_n itself, but occur before $3/2\ a_n$ does. It might be helpful to have one multiple which is by a power of 2 divided by a power of 3, and another which is by a power of 3 divided by a power of 2. Then, for different possible values of a_n (if they're large enough), one may find that one or other of these multiples will always produce an 'improvement'.

Cătălin Gherghe, who is in charge of the organising committee for this whole competition, as well as being the Romania A Team Leader, is sad and (needlessly) apologetic that there's **no farewell party** with which to finish and celebrate this Olympiad, featuring the traditional **ţuică** (Romanian plum spirit) and **pălincă** (fruit brandy), and giving us the chance to get to know people better. I concur.

Our team definitely valued and enjoyed this competition, but it would have been much better for them to go in person, I'm sure, had that been possible – which of course it wasn't.

There have been some videos of Romania – in particular the Buşteni area, and the nearby Peleş Castle – put onto the official website for students to enjoy, and also a lecture by Professor Mihăilescu on some of the maths behind his **solution to Catalan's Conjecture, showing that 8 and 9 are the only consecutive positive integers which are both perfect powers**. And there is also a graph theory lecture by one of the coordinators, Lucian Ţurea, who had himself as a student achieved a score of 40/40 in the 2007 Balkan Olympiad (which was held in Greece – I was actually there too).

I have many wonderful memories of my own from previous Balkan Olympiads: of a sense of adventure, and of great fun, of making new friends (and seeing old friends), of helpful guides looking after the teams and showing them around, of local TV interest, of many languages in use and of students standing on stage with their national flags. There have been grand host schools, (painted) monasteries, castles, museums, ancient ruins and caves; and in the spring sunshine we've had visits to botanical gardens, 'butterfly valleys' and beaches. There's been swimming, sailing, football, card games, discos, local food and drink, good spirits, jokes, late night arrivals in new cities, early morning problem selection meetings, late night marking, and partying, and so on, and so on…

So many of these memories of mine involved the relevant UK teams, and it is certainly a regret that I haven't physically seen this year's team, and as a result this report is rather sadly focused on just my experience of this Olympiad, and not really the team's experiences (or even Kasia's); my apologies.

Concluding thoughts

This year's UK students should all look back with a great deal of satisfaction on representing their country, and doing so very well, and I wish them the very best for their futures, mathematical and otherwise.
Thank you to Wilf, Ojas, Sean, Linus, Eleanor and Jenni for your efforts, and I hope to be able say this to you in person in the not too distant future.

Thank you too to Kasia for sharing the work of organising things, and for carefully reading through exam scripts and making the case for marks in the coordination sessions.
Also to James Gazet, Dominic Yeo, Kit Richardson and anyone else at UKMT who has helped.
And thank you to Loughborough Grammar School for giving me some time off work.

And a **very big thank you to the Romanian organisers** and coordinators for making this tournament work so well. In addition to people already mentioned, I should pick out **Radu Gologan**, the Chief Coordinator here – whom I first met at the 2005 Balkan Olympiad and most recently saw at the 2019 IMO when I was coordinating the Romanian team on one of the questions (he was also on the IMO Board) – and **Mihai Bălună**, who was the head of the Problem Selection Committee here.

Let's hope it won't be too long before mathematical discussions, spirits and friendships can flow at a 'real' Balkan Maths Olympiad again!