RMM 2019 – UK Report

Gabriel Gendler*

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Introduction

The eleventh Romanian Master of Mathematics was held in Bucharest between 20th and 25th February 2019. The competition is by invitation only, and these invitations are generally extended to countries with traditions of strong performance in the older and more inclusive International Mathematical Olympiad (IMO). We are therefore most grateful to the organisers who have honoured us with an invitation to each iteration of the event since its birth in 2008; we are also grateful to the UK Mathematics Trust[1], which has fostered a healthy culture of advanced mathematics competition training over the last few decades.

Although only four students can be entered for the team competition, the UK sends six students in order to mimic the experience of an IMO. Here they are:

Naomi Bazlov	King Edward VI HS for Girls, Birmingham	16
Alex Darby	Sutton Grammar School for Boys	18
Tom Hillman	St Albans School	17
Benedict Randall Shaw	Westminster School	16
Aron Thomas	Dame Alice Owen's School	17
Tommy Walker Mackay	Stretford Grammar School	16

My colleagues accompanying the UK delegation were deputy leader James Gazet (St. Paul's School) and Georgina Majury (Peterhouse, Cambridge). The reserves were Yuhka Machino (Millfield School) and Thomas Pelling (Bedford School).

Results and thanks

Here are the UK's results[2]. The medal boundaries were 37 for gold, 30 for silver and 24 for bronze.

*Email: gabriel.gendler@mail.huji.ac.il

[1] www.ukmt.org.uk

[2] The full set of individual and team results for the year can be found at rmms.lbi.ro/rmm2019/index.php?id=results_math

	Q1	Q2	Q3	Q4	Q5	Q6	$\sum$	
Naomi Bazlov	7	0	0	7	1	0	15	Honourable Mention
Alex Darby	7	0	2	7	2	0	18	Honourable Mention
Tom Hillman	7	7	1	7	3	0	25	Bronze Medal
Benedict Randall Shaw	7	0	1	7	1	4	20	Honourable Mention
Aron Thomas	7	7	1	7	4	0	26	Bronze Medal
Tommy Walker Mackay	7	0	1	-	-	-	8	Honourable Mention

Our sympathies to Tommy who was taken ill on the morning of the second exam and did not participate.

The RMM is an historically hard competition (more so than the IMO), and we do not pull our punches when selecting our six contestants. Therefore one would expect to see some of these names reappear when the UK hosts the IMO this summer. While the whole squad will be working hard between now and then at various camps and in correspondence training, the results above represent a solid foundation on which to build a strong performance for the home team.

Congratulations are due to the USA on winning the competition. The team score is the sum of the highest three scores out of the first four participants in a country's line up, and their three gold medals (a third of all golds awarded) made for an impressive tally. Korea, Serbia and Israel filled out the top places. The three new additions (Israel, Iran and Georgia) performed remarkably well, placing fourth, seventh and eighth respectively. Aside from the three Americans, gold medals went to two Russians, a Serb, a Romanian, Jan Fornal of Poland who uniquely also took gold last year, and the individual winner, Lior Hadassi of Israel, who scored all but one mark.

All of these remarkable mathematicians deserve to be celebrated, and it stands to reason that a country not half the size of ours which manages to muster the resources to organise a mathematics competition as prestigious and successful as this one on an annual basis must be a country proudly and deeply committed to cultivating and then celebrating generation after generation of elite mathematicians. Romania is the home of the IMO and of international competition mathematics, and the RMM is the crown jewel in an astounding national mathematics setup which gives so much to Romanian students and to the global mathematics community.

We are indebted to Radu Gologan for his tireless efforts organising the competition, and to Tudor Vianu National College of Computer Science, the rather extraordinary school which hosts us every year and whose walls are covered with lists of medallists in every international olympiad imaginable that would be the pride of any medium-sized country. We also thank Iulia Manicea, Ana-Maria Nadolo and all the organisers who were as always so understanding, responsive to our needs and quick to solve problems. Calin Popescu, Ilya Bogdanov and the problem selection committee produced wonderful papers three times, including once under enormous pressure, and Ilya was a patient and insightful guide as the jury navigated its various decisions, from the most routine to the completely extraordinary. Finally we found the quality of co-ordination first class, and we thank Mihai Baluna and his experienced and capable team.

The UK team owes thanks as always to Bev Detoeuf and the UKMT office who organise everything so well and make life so easy for the students and for the leaders. Special thanks also go to our extensive network of trainers who give up time in blocks of an hour, a day and sometimes a week to support the students as they train. In particular, the two linchpins of the whole operation, Dominic and Geoff, cannot ever be thanked enough.

If you should ever come upon an opportunity to go for a fun-filled holiday with two individuals of your choice, I recommend picking James and Georgina, who are wonderful company as comics, as mathematicians and as childcarers. If you also get to choose the six children you'll be accompanying, I recommend anyone but the vicious delinquents with whom we were burdened, and who attacked me with balloons at the closing gala. Aside from that they too were good company and talented mathematicians.

Contest Papers

Modelled after the IMO, the contest consists of two papers of three questions each, taken on consecutive days with four and a half hours allotted to each test. Seven marks are available for each question, so for the most talented and competitive students 42 really is the answer to life, the universe and everything.

Day 1

Problem 1. Amy and Bob play a game. At the beginning, Amy writes down a positive integer on the board. Then the players alternate moves. Bob moves first. On any move of his, Bob chooses a positive integer b, and replaces the number n on the board with $n - b^2$. On any move of hers, Amy chooses a positive integer k, and replaces the number n on the board with n^k. Bob wins if the number on the board ever becomes zero. Can Amy prevent Bob from winning?

Russia, Maxim Didin

Problem 2. Let $ABCD$ be an isosceles trapezium with AB parallel to DC. Let E be the midpoint of AC. Denote by Γ and Ω the circumcircles of triangles ABE and CDE, respectively. The tangent to Γ at A and the tangent to Ω at D intersect at P. Prove that PE is tangent to Ω.

(The trapezium $ABCD$ with AB parallel to DC is *isosceles* if $\angle BCD = \angle CDA$.)

Slovenia, Jakob Jurij Snoj

Problem 3. Given any positive real number ε, prove that for all but finitely many positive integers n, any simple graph on n vertices with at least $(1+\varepsilon)n$ edges has two different simple cycles of equal length.

(A *simple graph* is a set V of vertices, together with a set E of edges, where each edge in E is a set of two vertices of V. A *simple cycle of length* k is a set C of $k \geq 3$ distinct edges in E, such that there is a sequence of distinct vertices $v_1, v_2, \ldots, v_k$ such that for each $1 \leq i < k$, $\{v_i, v_{i+1}\}$ is in C, and $\{v_k, v_1\}$ is in C.)

Russia, Fedor Petrov

Day 2

Problem 4. Prove that for every positive integer n there exists a (not necessarily convex) polygon with no three collinear vertices, which admits exactly n different triangulations.

(A *triangulation* is a dissection of the polygon into triangles by interior diagonals which have no common interior points with each other nor with the sides of the polygon.)

Iran, Morteza Saghafian

Problem 5. Determine all functions $f\colon \mathbb{R} \to \mathbb{R}$ satisfying

$$f(x + yf(x)) + f(xy) = f(x) + f(2019y),$$

for all real numbers x and y.

Slovenia, Jakob Jurij Snoj

Problem 6. Find all pairs of integers (c, d), both greater than 1, such that the following holds:

For any monic polynomial Q of degree d with integer coefficients and for any prime $p > c(2c + 1)$, there exists a set S of at most $(\frac{2c-1}{2c+1})p$ integers, such that

$$\bigcup_{s \in S} \{s,\ Q(s),\ Q(Q(s)),\ Q(Q(Q(s))),\ \ldots\}$$

contains a complete residue system modulo p (i.e., intersects with every residue class modulo p).

Croatia, Adrian Beker

UK Team Diary

Wednesday 20th February

I wake at 2:30am. This is completely intolerable and I will be tired and in a foul mood throughout the competition as a result. After four uneventful hours I am at the gate and ready to board. Also at the gate are eight excitable and mathematical looking youths, along with three mathematical and responsible looking adults, so I assume that they are an RMM team. Since I am in Ben Gurion Airport I take it that they are the Israeli team. This is their first time participating in a growing competition and they are a welcome addition, what with their being seriously good at maths and endowed with four returning IMO medallists.

The flight and subsequent trip to the hotel are uneventful. I am about five hours ahead of the rest of our delegation so I go for lunch with the Israelis and then hurry back when I hear that the British are coming. They are in good spirits, having stayed by the airport and caught more sleep. We catch up over a walk and soft drinks, and then again over dinner. In the evening James and I receive the problems and the shortlist of spares, and work hard on all of them well into the night.

Thursday 21st February

In the morning the contestants are treated to an interesting lecture, which Tommy can tell you more about.

> *The lecture was on the subject of code theory. This is an area of pure maths inspired by the real-life problem of how to transmit information in such a way that if the information is damaged in some way (such as a CD being scratched) the original message can still be pieced together from what remains (the answer is to encode the information with "codewords" that are sufficiently different from each other not to be confused after information loss). To illustrate this, the lecturer (Cătălin Gherghe) played a CD that had been scratched after various levels of application of code theory to the information on it. It was barely possible to hear the music over the crackling on the first recording, but by the last recording you wouldn't know it was damaged at all. From that point onwards the lecture became slightly more theoretical (by which I mean that there was more maths and less listening to CDs). We were introduced to the Hamming inequality, which describes effectively how many different codewords you can have if you want them to have a certain length and not be too similar to each other. Then we were shown a combinatorics question from EGMO which could be solved quickly using the Hamming inequality, and the lecture was concluded shortly after this (assuming I didn't just stop taking notes).*

Meanwhile, the jury is convened and we spend the morning continuing to work on the problems. Around lunchtime, discussion begins and within minutes we agree to the papers precisely as proposed by the Problem Selection Committee.

The English language committee meets and does a thorough job of making the statements of the questions watertight. This is no mean feat – the graph theory question 3 requires some very careful definitions and for safety we think it wise to define isosceles in the context of a trapezium in question 2. Most important of all is replacing the erroneous term "trapezoid" with the correct "trapezium", with the full blessing of the American leaders Po-Shen Loh and Evan Chen, who have no reason to fear the wrath of Geoff Smith if their country's conventions of nomenclature are overlooked. We spend even longer on the second paper, ironing out tricky details that I will not address here. The full jury is happy with our work and after a unanimous vote the finalised paper is distributed and leaders with contestants whose first language is not English set about producing translations.

All this has been achieved at break-neck pace, so after the opening ceremony we will be able to head back to our hotel with the students and I will be able to sleep. This prospect means that I struggle to pay attention to the proceedings. Highlights include a rendition of all eleven verses of the Romanian national anthem by an impressive youth choir, and speeches from several key figures in Tudor Vianu: the principal, the chair of the parents' association and the former president of the student council, the focus of whose speech is what he considers to be a novel approach to finding oblique asymptotes. He ends with a rousing declaration that "Romania is yours for the taking". We exchange glances with our friends in the Hungarian team, whose Prime Minister might well have taken that as an invitation were he present.

Friday 22nd February

The students head into the first exam in good spirits. The Bulgarians are given tomorrow's paper (as well as the English language version of today's paper). Theirs are replaced and we agree that the entire second paper is now unfit for use, and set about assembling a brand new paper from the shortlist. This is challenging work and there is no clear best solution, but diplomacy and solidarity emerge victorious and after a long and exhausting day we have chosen problems, decided on their order, rewritten them in the ELC, translated them into other languages and set mark schemes for all six problems.

The students have emerged happy with what promise to be six perfect solutions to Question 1, as well either two or three for the geometry and lots of partial progress on the very difficult combinatorics problem. We debrief with them and at five o'clock we receive their papers and get to work marking. This is particularly challenging for me because I am temporarily unable to write, but James makes short work of the first two questions while I comb through the scraps in Question 3 and find some nice ideas which should be worth points.

Saturday 23rd February

All students receive the correct papers today and emerge satisfied that they had a good go at a very difficult set of problems. We are once again delighted with what ought to be full marks on Question 4, as well as some partial marks on the functional equation and a lot of good work by Benedict on the final question of the competition. Co-ordination goes smoothly, with our meetings for the first two questions taking less than thirty seconds combined. There is some sparring over 1s and 2s in the graph theory problem, but everything remains civil and the decisions at which we arrive seem correct.

The students are now free to kick back and enjoy Bucharest, a really beautiful city whose post-Soviet development means that its historic buildings are now much better matched by their modern surroundings than they were when I first came in 2013. Here's an account of what they get up to from Aron.

> *Being the default recreational activity for the UK team during maths competitions in Southeastern Europe, it was no surprise that we ended up in an escape room soon after the conclusion of the second exam. Set in a wine cellar and themed around a 1970s Romanian corruption scandal that was later adapted into a film, the escape room was notably different from others I have been to before, with us taking the roles of trapped investigative journalists. The experience certainly benefited from Romania's more relaxed approach towards health and safety compared to the UK due to the addition of slingshots, ladders and fairly high power lasers making the escape room more memorable than normal. Despite Ben and Alex's disappointment that escape could not be limited to a proper subset of the team, we all enjoyed the escape room, and managed to escape with lots of time to spare.*

I head back to the hotel and sleep all afternoon. We enjoy a celebratory dinner in a great restaurant round the corner. Then James and I return to today's scripts. I was not expecting to encounter such a mess for Question 4, but after a very late night I am convinced that we can ask for full marks. Benedict will be disappointed to lose at least a mark for bad algebra on question 6.

Sunday 24th February

The students get a well deserved lie in. I also deserve a lie in but James makes sure I'm up in time for our first co-ordination meeting. A tricky feature of co-ordination here is that rough paper is not photocopied for the co-ordinators, so they only see two marks worth of work for Benedict (but are bemused by his capitalised "SEE ROUGH PAGE 3" at the bottom of his second page and suspect that something is afoot). After a quick glance at this page they agree with our assessment of four marks. While no problems arose as a result of the policy this year as far as I know, I think it would be worth addressing in future; until then we will tell our students to default to answer paper at this competition. Question 4 goes off without a hitch, although sometimes I think a mark lost for lack of clarity might be a valuable lesson to some of these students about the importance of writing like a grown-up and not making your team leader stay up until a goodness-forsaken hour.

The jury convenes for the last time and agrees on sensible medal boundaries. The only boundary up for debate is the gold cut-off, which determines whether nine or fourteen students receive top honours. A gold medal at this competition is and ought to be a very special result, and we agree on the stricter boundary. Finally we agree to treat the lost second paper as under embargo like the rest of the shortlist (the Bulgarians are unfortunate not to be able to use these three problems for training).

Georgina continues to treat the team to further entertainment, this time involving an art gallery.

The closing ceremony is a happy and relaxed affair. We are treated to two performances from a local folk dance group, who are full of energy and whose outfits are a sight to behold. We also enjoy a cover of Despacito by Justin Bieber, rendered on the piano and classical guitar. The decision to call each medallist up individually costs time but feels right. This is a competition built on solidarity and friendship, and the opportunity to celebrate each other on an individual level with whooping, cheering, whistling and heckling is a fitting end to the contest.

We return to yesterday's restaurant for dinner with the victorious American team. Since the length of the table cannot be much greater than the length of the room, Georgina, James, Po-Shen and I sit at a smaller table next to the main group and discuss the cultures of competition, camp and university mathematics in our respective countries, as well as research interests. An interesting constraint when paying on multiple cards is that there must be a partition of the items on the receipt corresponding to the desired partition of the bill. We observe that this problem is NP-complete and Po-Shen recommends to our waiter that the students are put to work on it. The waiter smirks and tells us in about as many words that it will be trivial for him. Lo and behold he returns within seconds having solved the problem. A fitting reminder as we prepare to leave Romania of just how good at maths Romanians are.

Monday 25th February

I am out of the hotel before the rest of the delegation is up, and away to the airport along with the Israelis, with whom I will be flying again. I can delight in the remaining problems from the shortlist and a book of inequalities that was given as a gift to each team. I choose instead to delight in writing this report. How delightful.