

IMO 2019, Bath (UK) - UK team report

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The UK Maths Trust² organises competitions, mentoring and other enrichment activities for talented and enthusiastic school-aged mathematicians. One strand is a training programme for the country's top young problem-solvers to introduce them to challenging material, enjoyable in their own right, but also the focus of international competitions.

The International Mathematical Olympiad is the original and most prestigious such event, now in its 60th edition. Over a hundred countries send teams of up to six contestants. This year the competition was hosted in the UK, with the main student site at the University of Bath. Freddie Illingworth was our deputy leader; we organised the preparation programme through the year, assisted by a large number of volunteers, many of whom also attended the IMO in the capacity of guides, academic coordinators, and members of the problem selection committee.

Hosting an IMO is considerably more busy than merely attending an IMO, and time was not so available directly after the competition. Consequently this report is rather brief. We include a statement of the problems of IMO 2019, a brief discussion of the results within the context of the UK training programme, and a selection of Dominic's experiences at the competition following, for the first time, the leaders' side of the programme.

The problems of IMO 2019

Problem 1

Determine all functions $f : \mathbb{Z} \rightarrow \mathbb{Z}$ such that, for all integers a and b ,

$$f(2a) + 2f(b) = f(f(a + b)).$$

SOUTH AFRICA (LIAM BAKER)

Problem 2

In triangle ABC , point A_1 lies on side BC and point B_1 lies on side AC . Let P and Q be points on segments AA_1 and BB_1 , respectively, such that $PQ \parallel AB$. Let P_1 be a point on line PB_1 , such that B_1 lies strictly between P and P_1 , and $\angle PP_1C = \angle BAC$. Similarly, let Q_1 be a point on line QA_1 , such that A_1 lies strictly between Q and Q_1 , and $\angle CQ_1Q = \angle CBA$.

UKRAINE (ANTON TRYGUB)

Problem 3

A social network has 2019 users, some pairs of whom are friends. Whenever user A is friends with user B , user B is also friends with user A . Events of the following kind may happen repeatedly, one at a time:

Three users A , B , and C such that A is friends with both B and C , but B and C are not friends, change their friendship statuses such that B and C are now friends, but A is no longer friends with B , and no longer friends with C . All other friendship statuses are unchanged.

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Initially, 1010 users have 1009 friends each, and 1009 users have 1010 friends each. Prove that there exists a sequence of events after which each user is friends with at most one other user.

CROATIA (ADRIAN BEKER)

Problem 4

Find all pairs (k, n) of positive integers such that

$$k! = (2^n - 1)(2^n - 2)(2^n - 4) \cdots (2^n - 2^{n-1}).$$

EL SALVADOR (GABRIEL CHICAS REYES)

Problem 5

The Bank of Bath issues coins with an H on one side and a T on the other. Harry has n of these coins arranged in a line from left to right. He repeatedly performs the following operation: if there are exactly $k > 0$ coins showing H , then he turns over the k th coin from the left; otherwise, all coins show T and he stops. For example, if $n = 3$ the process starting with the configuration THT would be $THT \rightarrow HHT \rightarrow HTT \rightarrow TTT$, which stops after three operations.

- Show that, for each initial configuration, Harry stops after a finite number of operations.
- For each initial configuration C , let $L(C)$ be the number of operations before Harry stops. For example, $L(THT) = 3$ and $L(TTT) = 0$. Determine the average value of $L(C)$ over all 2^n possible initial configurations C .

USA (DAVID ALTIZIO)

Problem 6

Let I be the incentre of acute triangle ABC with $AB \neq AC$. The incircle ω of ABC is tangent to sides BC, CA , and AB at D, E , and F , respectively. The line through D perpendicular to EF meets ω at R . Line AR meets ω again at P . The circumcircles of triangle PCE and PBF meet again at Q .

Prove that lines DI and PQ meet on the line through A perpendicular to AI .

INDIA (ANANT MUDGAL)

Overall comments

I thought this was a good set of problems for an IMO. People will always find angles to complain about, but this represented a good range of styles, topics, and difficulty. In particular, it was good to see a very accessible Problem 1. The UK is fortunate to be able to offer its students a balanced and detailed preparation programme, building up towards regular success at the medium-level problems on an IMO. But for some countries, Problem 1 offered a route towards their students achieving a bronze medal or an Honourable Medal (for a perfect solution to one problem) which is often not available.

It was also pleasing to include a medium-level geometry question, last seen at an IMO in 2011. Very exotic geometry is not really my cup of tea, but I was impressed to see a range of solutions to Problem 6 appear during the contest, and I was a big fan of Problem 3, for which some of the ideas about decomposing graphs felt well linked to modern techniques in the field.

I also note that the six problems were written by authors from five different continents. Australia and New Zealand have also been regular contributors in recent years, and it's possible we may see six problems from six continents in a future IMO. It's pleasing that the 'I' in IMO is fully reflected in the submissions.

Background and UK results



Maximising our ranking at the IMO is not the unique focus of the programme, but in any case, UK came 20th at IMO 2019, with a full set of medals, including a gold medal for Liam, and we are very proud of the team for their results, and the hard work which led to them. Here are the individual results.

	Q1	Q2	Q3	Q4	Q5	Q6	Σ	
Alex Darby	4	0	0	7	7	0	18	Bronze Medal
Liam Hill	7	6	0	7	7	7	34	Gold Medal
Tom Hillman	7	6	0	5	7	0	25	Silver Medal
George Mears	7	6	0	7	2	0	22	Bronze Medal
Benedict Randall Shaw	7	7	1	0	7	0	22	Bronze Medal
Aron Thomas	7	6	1	7	7	0	28	Silver Medal

At the time there was much gnashing of teeth about the 1 mark penalty on Q2 arising from a subtlety of diagram dependency, but in the light of hindsight, all was handled sensibly. Ben can consider himself unfortunate to misread Q4, as he would have been well placed to solve the correct statement of this problem, and join Aron and Tom in earning a second IMO silver medal. We are all very impressed with Liam's analysis of Q6 via complex numbers, which earned him a well-deserved gold medal.

Ben and Aron both have one more year of school, and have excellent chances to aim high at IMO 2020. Alex, Liam, Tom, and George are now leaving school, and starting their university studies in Cambridge in October. We wish them, and all their Year 13 colleagues from our camps and mentoring programmes every success as they start the next step of their mathematical journeys.

Events at IMO 2019

Pre-IMO camp

The IMO adventure begins with a final residential training event, giving the students a chance to come together after a busy period of school exams, and re-focus on olympiad mathematics. After a last-minute change of venue we will be working at the University of Bath.

There are already signs of the impending IMO, which is arriving at the same site in a week from now. This will involve over 600 students from all over the world, but for now we are a group of thirteen, with the UK team joined by our first reserve, Yuka, and by the six members of the Australian team. On Monday, the Australians are more interested in the sleeping arrangements, but all the students are impressed by the range of options at the university's 'Lime Tree' canteen.

Each morning for the next few days the two teams will attempt a 4.5 hour exam. The Australian leaders Angelo and Sampson have done an excellent job of choosing some suitable problems, and these papers will serve as excellent preparation for the IMO itself. The UK team are in very safe hands with deputy leader Freddie Illingworth and geometry expert Sam Bealing to mark their work and give valuable feedback and guidance.

Leader site

This is my first experience of the leader role at an IMO, which involves leaving the students to attend a separate location in order to choose the problems for the IMO papers. This involves a short train under the Severn estuary to Newport, and the impressive Celtic Manor which looms above the city. This hotel is familiar with hosting international political summits, and major golf tournaments; and the arrival of 100 mathematicians from all around the world surely merits similar gravitas.

Rather than detail each day's activities, I will pick out some highlights.

Paper selection

The main task is to assess the *IMO shortlist*, a book of 32 problems chosen by the problem selection committee from over 200 submissions from participating countries. Some of the problems are intended to be very difficult, and there's not really enough time to make serious attempts at many of them before solutions are distributed. I attempted C5, and felt confident to speak in favour of its inclusion; and was pleased that this worked out as Problem 3. Many of the problem selection committee are former UK IMO participants, and it is interesting to hear their perspectives on the various problems.

The process for deciding on a paper involves very complicated sequences of voting. This is now electronic, chaired with élan and humour by Adam McBride, who performed the same role when UK hosted IMO 2002 in Glasgow. The physical voting paddles remain, as a way to get Adam's attention to offer a comment. Possibly I make too many such comments.

Many details are now lost in the mists of time, but a number of leaders are anxious about the possibility of including G1, since it feels inappropriately straightforward, and the American leader Po-Shen Loh has an effective strategy for eliminating it. Various harder problems come in and out of focus of popularity, though we are working in a fog of ignorance, given the lack of time to engage seriously. As far as I recall, A7, C7, and N8 were all in contention at times, but then waned. We'll look forward to using these as part of next year's preparation as all seemed interesting.

Wording

It is essential that the English version of the paper is worded unambiguously and concisely. When writing research papers, there is normally a broader context that helps prevent ambiguities arising, and I'm sure I'm not the only person who regularly exchanges eg 'set' and 'collection' to move L^AT_EX's automated linebreak into a better place. There is no such luxury here, as it must be possible for all 600 students to interpret every question perfectly, and the English will be used by other leaders to translate into their own languages. This is quite challenging, especially for the complex premises of Problems 3 and 5.

Our Russian colleague Pavel Kozhevnikov raises a nuanced aspect of diagram dependency in Problem 2. Like Cassandra, his prophecy is valid but passed over, and like the Trojans we allow danger to sneak onto IMO Day One, via the intersection of not-necessarily-non-parallel lines, which will open up to reveal an army of 6/7s later.

I don't envy other leaders who now must produce similarly tightly-worded drafts in their own languages without such oversight and discussion. However, the timings are opportune, as this afternoon of freedom for the English-speakers coincides with a Quite Important cricket match.

Q & A

One of the major differences between the student site and the leader site is the inertia of the period when the exams are taking place. Over in Bath, there would have been plenty of nerves, with a carefully-coordinated schedule building up to 600 students starting the exam simultaneously. Whereas at Celtic Manor, I am excited for our team engaging with the problems, but my only task is saunter down from a later breakfast to see whether any queries are raised during the initial 30 minute Q&A period of the papers. This is again handled electronically, and goes smoothly, though some countries' students ask questions about the meaning of 'function' that makes us anxious that they may not be well-equipped to attempt Problem One, which would be a shame.

Communication with the UK team or with Freddie is not permitted. Over in Bath, the epic sequence of scanners are whirring into action, so we must wait to find out how our students found the paper. There is time to enjoy some of Celtic Manor's other diversions. The resort has hosted major golf tournaments, including the 2010 Ryder Cup, but probably hasn't ever seen as fierce a competition as what unfolded on its 36 hole mini-golf complex with the as-yet-unoccupied Day Two coordinators.

Post-exam excursions

It is traditional that IMO contestants are taken on excursions, while the adults are busy with marking and coordination of the results. In 2017, the UK team had the chance to visit Copacabana beach and *Cristo Redentor* in Rio after the IMO papers were concluded. In 2015, the deputy leaders even had the opportunity to ride an elephant in Chiang Mai. It's easy to deploy classical British self-deprecation, but my impression was that the Brazilian and Thai teams enjoyed their excursions to Stonehenge and Salisbury just as much as we enjoyed their hospitality in previous years. I think even the British team enjoyed seeing some familiar icons in a new context, despite the predictably unpredictable weather.

I think everyone also enjoyed the flexibility on these two days, and a number of international teams took the chance to visit London with their guides. Those who stayed near Bath had the chance to use the university's excellent sports facilities. Returning from coordination, I caught

the dramatic final moments of the UK team's 5-a-side football match, featuring extreme heroism from Alex Darby in goal, ensuring a draw against Portuguese/Israeli composite opponents.

The post-exam programme concludes in the impressive Forum Theatre in central Bath with the IMO lecture, delivered by Ben Green, drawing links between some notable IMO problems and contemporary directions in research. With the pre-IMO camp leading straight into the IMO itself, the UK team have now experienced about thirty consecutive meals at the Lime Tree, and are grateful for the opportunity this now affords for a variation. We also hear more candid details about the student experience of IMO 2019 - they seem impressed at disco lighting features available in all their bedrooms, and by the levels of administrative detail paid to bathroom breaks during the IMO papers.

Farewell celebrations

Someone who has attended a double-figure number of weddings this summer might find the giant marquee which hosts the closing ceremony to be familiar. A small number of speeches are made, but none are ribald, and no one cries. A number of long-standing UKMT volunteers get starring roles in distributing the prizes to the high-scoring participants, including Liam's gold medal.

This leaves plenty of time to enjoy the other activities on offer, essentially a small fairground. While many teams queue for the ferris wheel, the UK and Australian boys extend their rivalries from the exam room to the dodgem arena. I am placed under extreme pressure to join on the flying chair carousel. I consent but do not enjoy this at all. By contrast, witnessing my reaction at close quarters seems to be one of the highlights of Aron's life so far.

The golden microphone

is a prize awarded to the leader who makes the most speeches. Regrettably, despite stiff competition from Dan Carmon of Israel, I am declared the winner. The trophy is genuinely elegant and will have pride of place on my bookcase next to the UNK voting paddle that played its role in summoning this victory, but I aim for this to be a one-off triumph.

UK Mascots

One extra member of the group is a plush jaguar cub, kindly gifted as a team mascot by Tom Hillman. At the winter camp in Hungary, a jaguar naming committee was assembled. The committee dismissed James, Joseph, Jacob, Joe, Jeremy, and José for various reasons. I proposed Keith in jest, but this was accepted. He has now been provided with a very small UK IMO t-shirt, and has an expression of glassy contentment about this situation.

While in Bath, a rival mascot is purchased by George. He is named Derek and is a pig, at least topologically. He is, however, almost spherical, which leads to difficulties when Ben tries to equip him with a UK IMO t-shirt. In the stakes to be the long-term UK IMO mascot, Derek has a small functionality advantage, since he also operates as a bank, but a significant location disadvantage, since he has subsequently moved to university with George.

Conclusion

Preparing a UK team for an IMO requires a huge amount of effort from a large number of people. Thanks are particularly due to:

- All the staff at our camps in Oxford, Hungary, Cambridge and Tonbridge. Nothing can quite compare to getting so many teenagers with the same interest in the same places, and helping them improve together, and we couldn't do this without all the volunteers happy to give up their time to support this.
- Thanks also to all the staff at the UKMT office, especially Bev Detoef, who has arranged everything for the IMO trips and our other events so tirelessly over the years.
- This year, the organisers of IMO 2019 overlap heavily with our own mathematical colleagues. The event went very smoothly, and was a wonderful experience for the hundreds of children from over 100 countries who participated. It's particularly worth thanking Geoff Smith, who laid many of the foundations for the UK's plans to host, in addition to continuing his role as IMO President, and Ceri Fiddes, who directed this complex, multi-dimensional event with great care for the details, and consideration for the students' welfare and enjoyment. I'm sure they will both enjoy catching up on sleep!
- Freddie Illingworth, for his invaluable assistance with marking and energising the students' academic preparation, as well as looking after everyone at the pre-IMO camp; and Sam Bealing for his injection of academic expertise, especially on the geometry front.



- Finally, of course, our UK team comprising Alex, Liam, Tom, George, Ben, and Aron. They all prepared for the competition with energy and care, and we've appreciated their good company, and enjoyed watching their mathematical progress.