# Balkan Mathematical Olympiad 2019 – UK report
# Chişinău, Republic of Moldova

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30th June – 5th May 2019

The Balkan Mathematical Olympiad is an annual competition for secondary school students. Eleven countries in Eastern Europe send official teams, and a generous handful of other nations participate as guests. The 2019 competition was hosted by the Republic of Moldova in Chişinău.

The UK has a self-imposed rule that no student may attend the BaMO[1] more than once. This rule ensures that more students get a chance to experience an international contest.

The UK team for 2019 consisted of:

| | |
|---|---|
| Brian Davies | St Edward's College, Liverpool |
| Thomas Finn | Bexhill College |
| Liam Hill | Gosforth Academy |
| George Mears | George Abbot School |
| Alevtina Studenikina | Cheltenham Ladies' College |
| Patrick Winter | Barton Peveril College |

Overall the event was a huge success, and would not have been without the hard work of a very large number of people. Thanks are due to the team members, their families and their teachers, and also to many people involved with the UKMT, particularly Geoff Smith and Bev Detoeuf. The team was accompanied by Ava Yeo and myself. Dominic Yeo joined the competition for two intensive days to help with marking and coordinating the scripts.

I would also like to express my gratitude to our hosts for an extremely well run and enjoyable competition. The organising committee, problem selection committee, coordinators and volunteers were all unfailingly helpful and professional, and deserve congratulations and thanks. Valeriu Guţu, Anatolie Topală and Valeriu Baltag all worked tirelessly throughout the week, as did many others, and all their guests were very well looked after. The Lyceum Da Vinci provided a superb venue, and the care they took over all aspects of our visit, particularly the UK team's dietary requirements, is very much appreciated.

The results of the UK team were as follows:

| | P1 | P2 | P3 | P4 | Σ | |
|---|---|---|---|---|---|---|
| Brian Davies | 10 | 0 | 10 | 0 | 20 | Bronze |
| Thomas Finn | 10 | 10 | 2 | 0 | 22 | Bronze |
| Liam Hill | 10 | 4 | 10 | 0 | 24 | Bronze |
| George Mears | 10 | 4 | 10 | 0 | 24 | Bronze |
| Alevtina Studenikina | 4 | 5 | 10 | 0 | 19 | Bronze |
| Patrick Winter | 10 | 1 | 3 | 0 | 14 | Hon. Mention |

The medal boundaries were Gold: $\Sigma \geq 31$, Silver: $\Sigma \geq 27$, Bronze: $\Sigma \geq 15$.

Technically the BaMO is an individual competition, but the countries with the highest aggregate scores out of a possible 240 were Serbia (182), Romania (180) and Turkey (164). The UK's total of 123 placed us ninth overall. Congratulations to Serbia and to the two students who scored full marks (both from Serbia).

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[1]The 'a' is not standard, but avoids confusion with the British Mathematical Olympiad.

## The problems

**Problem 1.** (*Proposed by Albania.*)

Let $\mathbb{P}$ be the set of all prime numbers. Find all functions $f : \mathbb{P} \to \mathbb{P}$ such that

$$f(p)^{f(q)} + q^p = f(q)^{f(p)} + p^q$$

holds for all $p, q \in \mathbb{P}$.

**Problem 2.** (*Proposed by Romania.*)

Let $a, b, c$ be real numbers, such that $0 \leq a \leq b \leq c$ and $a + b + c = ab + bc + ca > 0$. Prove that $\sqrt{bc}(a + 1) \geq 2$. Find all triples $(a, b, c)$ for which equality holds.

**Problem 3.** (*Proposed by Greece.*)

Let $ABC$ be an acute scalene triangle. Let $X$ and $Y$ be two distinct interior points of the segment $BC$ such that $\angle CAX = \angle YAB$. Suppose that:

1) $K$ and $S$ are the feet of perpendiculars from $B$ to the lines $AX$ and $AY$ respectively;

2) $T$ and $L$ are the feet of perpendiculars from $C$ to the lines $AX$ and $AY$ respectively.

Prove that $KL$ and $ST$ intersect on the line $BC$.

**Problem 4.** (*Proposed by Cyprus.*)

A grid consists of all points of the form $(m, n)$ where $m$ and $n$ are integers with $|m| \leqslant 2019$, $|n| \leqslant 2019$ and $|m| + |n| < 4038$. We call the points $(m, n)$ of the grid with either $|m| = 2019$ or $|n| = 2019$ the *boundary points*. The four lines $x = \pm 2019$ and $y = \pm 2019$ are called *boundary lines*. Two points in the grid are called *neighbours* if the distance between them is equal to 1.

Anna and Bob play a game on this grid.

Anna starts with a token at the point $(0, 0)$. They take turns, with Bob playing first.

1) On each of his turns, Bob *deletes* at most two boundary points on each boundary line.

2) On each of her turns, Anna makes exactly three *steps*, where a *step* consists of moving her token from its current point to any neighbouring point which has not been deleted.

As soon as Anna places her token on some boundary point which has not been deleted, the game is over and Anna wins.

Does Anna have a winning strategy?

**Time allowed: 4 hours and 30 minutes.**
**Each problem is worth 10 points.**

## Solutions

Official solutions can be found at `http://bmo2019.md/problems`. The solutions below differ in a few places (notably Q3 and Q4) and aim to give some indication of how the solutions might have been discovered.

## Problem 1

### Initial thoughts

(i) The answer must be $f(p) = p$ for all $p$ only. The idea that some other function from primes to primes might obey this kind of algebraic relation is genuinely incredible.

(ii) Since we are working with the set of primes, we might want to use Fermat's little theorem, and the Lifting the Exponent Lemma. Perhaps we will end up showing that $f(p) - p$ has infinitely many prime factors and must therefore be zero.

### Solution

We start by plugging in some values.

The easiest pair is $(p, q) = (2, 3)$ which tells us that $f(2)^{f(3)} + 9 = f(3)^{f(2)} + 8$.

At this point we might rearrange to $f(3)^{f(2)} - f(2)^{f(3)} = 1$ and invoke Mihăilescu's theorem[2] to conclude that $f(2) = 2$ and $f(3) = 3$. However, the sledgehammer here seems out of all proportion to the nut in this case.

Instead we might notice that since 9 and 8 have opposite parities, it must be the case that $f(2)$ and $f(3)$ also do. This is useful since even primes are in mercifully short supply. If $f(2) \neq 2$ then $f(3) = 2$, but the same argument then also shows that $f(q) = 2$ for all odd primes $q$. This seems implausible and taking any specific pair of odd primes is enough to reach a contradiction. However, we can be a little more general.

If $f(p) = f(q)$ then the awkward $f(p)^{f(q)}$ and $f(q)^{f(p)}$ terms in the original equation cancel to give $p^q = q^p$. Since we are working with primes this is enough to show that $p = q$. We conclude that $f$ is injective and thus that $f(2) = 2$.

So far we have $2^{f(q)} + q^2 = f(q)^2 + 2^q$ or, getting all the $f$s on one side,

$$2^{f(q)} - f(q)^2 = 2^q - q^2 \text{ for all } q \in \mathbb{P}.$$

Let us pause and ponder.

For big values of $q$ and $f(q)$ the terms $2^q$ and $2^{f(q)}$ will dominate here so if they are not equal one side of the equation will be larger than the other. (Since $f$ is injective $q$ and $f(q)$ will certainly eventually both be big.)

Noticing this and deciding to pursue it requires considerable composure, not least because we are itching to use the fun facts about primes listed in (ii). Daring to hope that $2^{f(q)} - f(q)^2 = 2^q - q^2$ is enough to force $f(q) = q$ requires some courage. It turns out that courage and composure are good traits in a maths contestants as well as the SAS.

We can define the function $g(n) = 2^n - n^2$ and note that $g(3) = -1$, $g(4) = 0$, $g(5) = 7$ and so on. A routine induction shows that $g$ is increasing, and therefore injective, if defined on integers greater than 2. This is enough to complete the solution to the problem.

It could be argued that this is not really number theory at all, but algebra. In particular if the odd prime numbers are replaced with any other set of odd numbers, the solution goes through in much the same way. All we are really using is parity and inequalities.

[2] Known as the Catalan conjecture until 2005, the theorem states that 8 and 9 are the only (non-trivial) integer powers which differ by 1.

## Problem 2

### Initial thoughts

(i) Each variable only occurs once in the target inequality, and the condition isn't too complicated. Maybe eliminating $a$ is worth trying.

(ii) The $\sqrt{bc}$ suggests AM-GM on $b$ and $c$ may help. Also the expressions $ab + bc + ca$ and $a + b + c$ in the condition reminiscent of the identities $(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$ and also $(a + 1)(b + 1)(c + 1) = abc + ab + bc + ca + a + b + c + 1$ so there may be mileage there.

### Solutions

Pursuing the first idea we write $a = \dfrac{b + c - bc}{b + c - 1}$ so the target inequality becomes

$$\sqrt{bc}\left(\frac{b + c - bc}{b + c - 1} + 1\right) \geq 2.$$

The $\sqrt{bc}$ is unpleasant, but we should pause before recklessly squaring everything. We are down to two variables, can we get to one? Well, we know that $b+c \geq 2\sqrt{bc}$ and if we could replace both instances of $b + c$ with $2\sqrt{bc}$ we would have a function of $\sqrt{bc}$ only. The trouble is that doing so would decrease both the top and the bottom of a fraction, so the fraction itself may go up or down. The solution is to 'do the division' in the fraction to reduce the number of instances of $b + c$ to 1 and regain some control.

We write the left hand side of our target as $\sqrt{bc}\left(2 - \dfrac{bc - 1}{b + c - 1}\right)$. If we replace $b + c$ by $2\sqrt{bc}$ the denominator of the fraction will get smaller, so the fraction will get larger, so the quantity under consideration will get smaller, which is the right direction for us. This seems good, but we need to be careful. The argument just outlined only works if the top of the fraction is positive, so we need to establish that $bc \geq 1$.

At this point we need to look at our other ideas. We know $(a+b+c)^2 = a^2+b^2+c^2+2(ab+bc+ca) > 3(ab+bc+ca)$ by AM-GM. We are given that $a+b+c = ab+bc+ca = k > 0$ which gives $k > 3$. Since $bc$ is the largest term in $ab + bc + ca$ we can conclude that $bc \geq 1$ and return to the previous line of attack.

Our aim is to prove that $\sqrt{bc}\left(2 - \dfrac{bc - 1}{2\sqrt{bc} - 1}\right) \geq 2$ so it suffices to show that

$(2\sqrt{bc} - 1)\left(\sqrt{bc}\left(2 - \dfrac{bc - 1}{2\sqrt{bc} - 1}\right) - 2\right)$ is non-negative.

Setting $x = \sqrt{bc}$ and simplifying, this becomes:

$$2x(2x - 1) - x^3 + x - 2(2x - 1) = -x^3 + 4x^2 - 5x + 2 = (x - 1)^2(2 - x)$$

We know $x \geq 1$ so we are done provide $x \leq 2$.

However if $\sqrt{bc} > 2$ then $\sqrt{bc}(a + 1)$ is clearly greater than 2 since $a$ is non-negative.

For the equality cases we need $b = c$ since we used AM-GM. The cases $\sqrt{bc} = 1$ and $\sqrt{bc} = 2$ give the triples $(1, 1, 1)$ and $(0, 2, 2)$ respectively.

A rather different solution can be found by adapting the last of our initial thoughts.

The condition shows that $(a - 1)(b - 1)(c - 1) = abc - 1$. It is also clear that the condition cannot hold if $a > 1$ or $c < 1$ so the sign of $abc - 1$ depends only on whether $b > 1$.

If $b \leq 1$ we have $abc \geq 1$. Now we consider $\sqrt{bc}(a + 1) > 2\sqrt{abc}$ by AM-GM on $a$ and 1. This implies the result.

The case $b > 1$ is a little harder. The target inequality is equivalent to $bc(a + 1) \geq 2\sqrt{bc}$ so, using AM-GM, it is certainly enough to prove that $abc + bc \geq b + c$.

If we consider $abc + bc - b - c$ and add $0 = a + b + c - ab - bc - ca$ we obtain

$$abc - ab - ac + a = a(bc - b - c + 1) = a(b - 1)(c - 1).$$

This is clearly non-negative since we are assuming $b > 1$. Chasing down the equality cases is straightforward, so we are done.

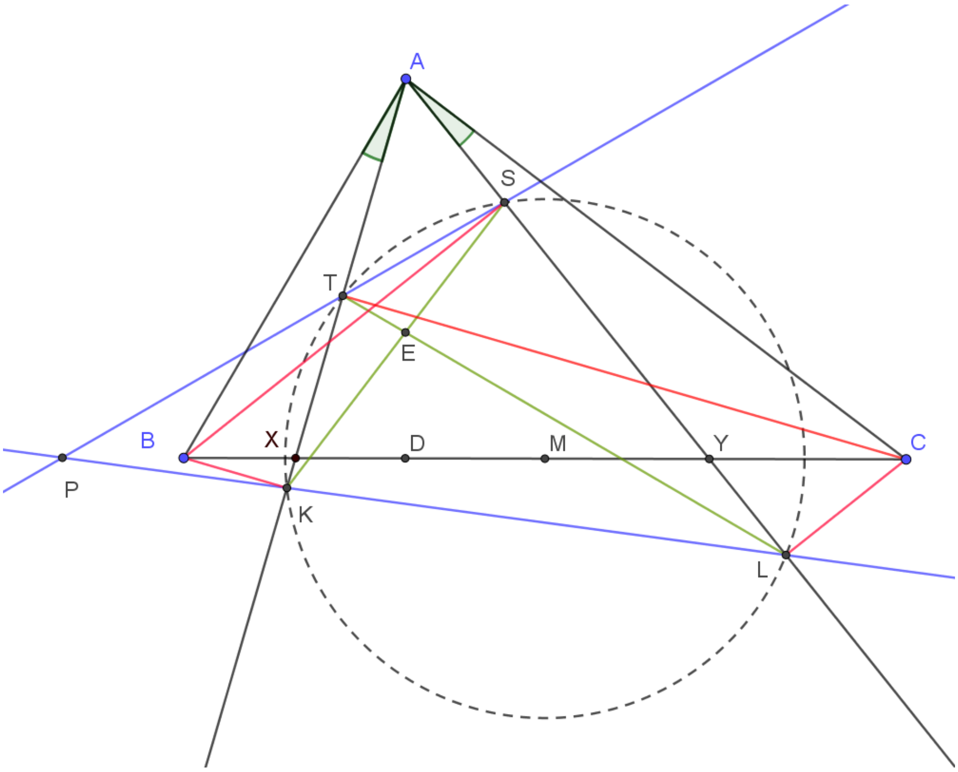
## Problem 3

### Initial thoughts

(i) There are only four points and one angle defined apart from the reference triangle, and lots of right angles – a calculation based approach may well be quite quick here.

(ii) The diagram suggests $KLST$ may be cyclic. If it is then we may be able to prove the currency using radical axes. Also, if we complete the quadrilateral then one point is $A$ and another is the point we want to show is on $BC$. Some facts about complete cyclic quadrilaterals may finish this off quite quickly.

### Solution




Our synthetic solution starts be establishing that $KLST$ is indeed cyclic. The right angles given in the question make it clear that $ABKS$ and $ACLT$ are cyclic so we have the chain of equalities $\angle SKA = \angle SBA = \angle ACT = \angle ALT$, where the middle equality uses the equal angle condition in the question.

We let $ST$ and $KL$ meet at $P$. Following point (ii) above we also let $KS$ and $TL$ meet at $E$ so that we have all 7 points of the complete cyclic quadrilateral $KSLT$.

Brocard's theorem now tells us that the centre of the circle $KSLT$ is the orthocentre of triangle $AEP$. In particular, if $O$ is the centre of $KSLT$, then $PO$ is perpendicular to $AE$. This opens up a path to the solution. It seems plausible that (i) $O$ lies on $BC$ and (ii) that the line $AE$ is perpendicular to $BC$ . If we can establish these facts, then the line through $O$ perpendicular to $AE$ is both the altitude of $AEP$ and the side $BC$ proving that $BC$ contains $P$.

For (i) we bravely conjecture that the midpoint $M$ of $BC$ may be the centre of circle $KSLT$. We consider the perpendicular bisector of $KT$, this is parallel $BK$ and $CT$, and exactly half way between them. Thus is intersects $BC$ at its midpoint. (We could make this observation sound grander by talking about the orthogonal projection sending $KT$ to $BC$.) The perpendicular bisector of $SL$ also passes through $M$ for a similar reason, so $M$ is the centre of $KLST$ as we had hoped.

For (ii) we introduce $D$, the foot of the altitude from $A$ to $BC$. The right angles mean that $D$ lies on both the circles $ABKS$ and $CATL$. If we consider these two circles, together with circle $KSLT$, the three radical axes are $AD$, $SK$ and $LT$. These concur at the point $E$, so we are done.

There are also a number of accessible calculation based approaches. Menalaus' Theorem on triangle $AXY$, together with some trigonometry is quite quick, as is cartesian coordinates with the axes chosen to be the angle bisectors through $A$.

## Problem 4

### Initial thoughts

(i) Since Bob can delete points from each boundary line each turn, we may restrict our attention to the top boundary line. If Bob can prevent Anna from reaching that, then she cannot win.

(ii) Anna moves three steps at a time, so it may be significant that 2019 is a multiple of 3.

(iii) By the time Anna reaches the top line, Bob will have had time to delete about a third of the points on the line, perhaps it is worth deleting every third point to preserve some symmetry.

### Solution

We will work on the grid defined by $|x|, |y| \leq 3N$, and describe a strategy for Bob which prevents Anna from reaching $y = 3N$.

If her $y$ coordinate is $3N - k$, we will say that Anna is $k$ *away* from the edge. If Anna is at $(x, y)$ we will call $(x, 3N)$ the *point ahead* of her. If we add $(x \pm 1, 3N)$ we obtain the *3 points ahead* and if we also add $(x \pm 2, 3N)$ we have the *5 points ahead.* It is clear that if Anna ends a turn 1 away from the edge, then Bob must ensure that the 5 points ahead are deleted by the end of his turn, or Anna will win. Similarly if she ends a turn 2 away, he must ensure the 3 points ahead are deleted, and if she ends a turn 3 away he must ensure the point ahead is deleted.

The last of these conditions is trivial, but deleting the 3 points ahead is only possible if at least one is deleted before Alice arrives. This observation, together with point (iii) above, suggests the following idea to Bob. He will start by deleting every third point on the boundary line. *If* he manages to do this before Anna gets close to the edge, then he can use his subsequent turns to ensure that the 3 points ahead of Alice are always deleted before her next turn. We will call this situation the case when Anna is *slow* to get to the edge. (We will make this precise in a moment.)

If Anna is slow, then she will never be able to win without first landing on a square which is one away from the edge. From here she will win unless Bob can make sure the 5 points ahead are all deleted before her next move. Even with every third point already deleted (thanks to Anna's slowness) this sounds like a tall order, but all is not lost. Let us suppose that the first time Anna ends a turn 1 away from the edge, she lands on $Q = (x, 3N - 1)$. She must have come from a point $P$ which was one of the following: $(x, 3N - 4), (x, 3N - 2), (x \pm 1, 3N - 3)$ or $(x \pm 2, 3N - 2)$. In each case we may assume that the three points ahead of $P$ were deleted before Anna moved from $P$ to $Q$. In the first three cases, all of 3 points ahead of $P$ are contained in the 5 points ahead of $Q$, so Bob can easily finish the job. In the final case he has to worry about three of the 5 points ahead of $Q$, but these are consecutive, so one has already been deleted thanks (again) to Anna's slowness.

The only other way Anna can reach $Q = (x, 3N - 1)$ is moving sideways from $P = (x \pm 3, 3N - 1)$. However this is also easy. We have seen that the first time Anna ends up 1 away, Bob can respond by blocking the 5 points ahead, so we may assume that when Anna moves from $P$ to $Q$ the 5 points ahead of $P$ are already deleted. This leaves 3 points for Bob to worry about, but, as before, one is already gone by our slowness assumption.

All this begs the question 'How slow is slow?'. For the arguments above to work, Bob would like to delete every third point on the top row *before* Anna first reaches a point that is at most 4 away from the edge. He can use his first $N - 1$ turns to delete the points with $x$-coordinates $\pm 3, \pm 6, \ldots, \pm 3(N - 1)$. After Anna's next turn she will have made a total of $3N - 3$ steps. If more than one of these is anything other than straight up then she is *slow.* Her $y$ coordinate will be less than $3N - 4$ and Bob can use his $N$th turn to delete only $(0, 3N)$ before trapping her as described.

It remains to describe Bob's strategy if Anna ends turn $N - 1$ on one of the points $(\pm 1, 3N - 4)$ or $(0, 3N)$. In the first case he deletes $(0, 3N)$ and the point ahead of Anna. In the second case $(0, 3N)$ *is* the point ahead so he deletes this and either of its neighbours. Bob has now deleted every third point, so if Anna does anything other than move to a point 1 away from the edge, then it is just as if she had been slow (so she will lose).

In the only remaining case Anna moves to $(\pm 1, 3N - 1)$. The 5 points ahead of her contain two of the points $(-3, 3N), (0, 3N)$ and $(3, 3N)$ as well as one other point just deleted by Bob. This means he can delete the remaining two points under threat, and Anna cannot win that turn. From now on, however, the arguments used for the case when Anna was slow all go through as before, so she is sure to lose.

It is worth noting that Anna can actually win on all other sizes of grid, and that she can win on all grids if she is permitted to make *up to* three steps per turn. These statements are easier than what we have just done, and are left as an exercise...

## Leader's Diary

### Tuesday 30th April

The team assemble at Stansted a little after midday and clear security with enough time for an unhurried lunch before boarding our flight. When we land we are greeted by a considerable group of volunteers from the Lyceum Da Vinci, the host school of the competition. We are photographed, encouraged to eat the delicious Moldovan apples on offer and generally made to feel very welcome. A minibus drops Ava and the students at the Lyceum, and I am taken to join the other leaders. Once we have eaten, it is time to start looking at the shortlisted problems, but I need a room to do so. It transpires that the leaders are to be accommodated in two different hotels, and I initially end up at the wrong one. The hotels are only a few kilometers apart and I am soon offered a lift. It is most definitely dark by now and the headlights of our car seem temperamental: occasionally they cut out entirely, and occasionally we meet rapid oncoming traffic. Fortunately the occasions do not coincide, and soon I am in my room (an individual log cabin on stilts) with the list of twenty proposed problems. I stay up a little later than planned since a combinatorics problem about an idiosyncratic motorist called Vlad is both irresistible and recalcitrant. I admit defeat just after 1am and reluctantly make plans for another 90 minutes work before breakfast at 7.30.

### Wednesday 1st May

The Jury meets punctually at 8.30am to start work on the paper. Our first task is to classify the problems according their difficulty, and eliminate any that are well known. In an ideal world we would all have attempted each problem from scratch before this meeting, but, as usual, the time constraints have prevented most (all?) of us from doing this. As a result there is a surprisingly strong correlation between the number of lines in the printed solutions and the eventual difficulty ratings.

Before we can finish classifying, it is time to head over to the opening ceremony which turns out to be a well-judged and well-organised affair. There are a number of short speeches interspersed with music and dance. The whole thing is hosted by a pair of students whose enthusiasm is unrelenting and threatens to overwhelm them each time they welcome us to 'THE REPUBLIC OF MOLDOVA'. The acts are excellent: a professional string sextet, a teenage modern dance group with an impressively athletic routine, and a younger folk dancing group in immaculate national costume all perform to a very high standard. In my view the show is stolen by the folk singing group, whose mean age I estimate at $\frac{9}{2}$ years.

To avoid contact with our students, which might compromise the security of the shortlist, we are ushered out early and whisked back to the hotel to finish our discussion of the geometry. We vote to eliminate a couple of unsuitable problems and start on the business of selecting the problems. We swiftly agree that the first question on the paper should be number theory. This is uncontroversial, since there are only two number theory problems to choose from and both are easy (by the standards of international maths competitions). Next we turn to the final question. We have no medium combinatorics so, with the easy slot spoken for, hard combinatorics looks likely. There are two options and both will be tricky to word unambiguously. In the end we decide against Vlad and his car, and opt for Anna and Bob instead. Various combinations of algebra and geometry are proposed for the middle two questions and a smooth process of single elimination voting gets us a paper by early afternoon.

As the unique native English speaker present, I am expected to play a major role in polishing the official English wording of the problems, but most other leaders are able and willing to express views on the matter as well. The geometry takes a little while: we decide to avoid the word isogonal and to add the constraint that the triangle should be scalene (this has the desirable effect of making the statement we want students to prove a true one). Then we move to the combinatorics and the fun really starts. Several hours later we have had enough of that particular brand of fun. I fend off some eleventh hour calls for a complete rewrite of the problem, and the Jury approves the official version just in time for dinner.

Next the other leaders need to translate the paper into their languages before they are approved and printed. Alevtina has apparently requested a Russian language version as well as an English one. Fortunately Kazakhstan is another regular guest nation, so the request poses no problem. I spend the time finding an alternative solution to geometry using harmonic pencils and then deciding that this is sophistry rather than progress. Everything is wrapped up by 11pm and I return to my cosy cabin.

**Thursday 2nd May**

Today is competition day. After breakfast we move to the Lyceum where the paper will be sat. Dominic Yeo has flown in at a thoroughly uncivilised hour to help marking and coordination and is waiting when I arrive.

Students may ask questions during the first half an hour and responses must be approved by the Jury. There are few straightforward questions and then a student asks 'May I use Mihăilescu's proof of the Catalan conjecture?'. He receives the response 'Any well-known theorem may be used, provided it is correctly stated.'. Like Gollum, this question will have its part to play, for good or ill.

After the Q and A the Jury rapidly approve the mark schemes prepared by the problem selection committee. Dominic suggests a few amendments to the geometry scheme which are adopted along with some other minor changes, but the brevity of the meeting does the organisers great credit. Next we are taken on a tour of the Cricova Winery. The description 'underground city of wine' is scarcely an overstatement. The complex has over 100km of tunnels and stores in the region of 20,000,000 litres of wine at any given point in time. Our tour includes discussion of the traditional method for making sparkling wine, a good deal driving through the tunnel network and an entertaining promotional video. It concludes with an opportunity to try (homeopathically small quantities of) the winery's produce – we fail to make it back in time for the end of the exam.

We rejoin the team towards the end of lunch and get their initial reactions to the paper. Clearly the distraction of the prime numbers in Q1 has led some people off the shortest path to a solution, and Liam has not been able to resist inversion in his solution to Q3, so coordination may prove interesting. However, everyone has solved at least one problem, and, provided some of the more exotic solution methods hold water, it seems likely to be a pretty good year for the UK. No one has solved problem 4, but this seems to be true of every team we speak to.

At this point I check into the students' luxurious 5-star hotel, complete with gym, spa and rooftop sushi restaurant. (We do not take advantage of any of these, but their presence sets the tone.)

The team's guide Valeriu takes them to the national museum of art while Dominic Y and I decamp to one of Chişinău's many excellent cafes to discuss the paper. As I try to share my partially formed thoughts on Q4, it becomes clear that writing up a complete solution under time pressure would be a formidable task for anyone. The number of near perfect scores will probably be small this year.

By 6.15pm the scripts have all been photocopied and Dominic and I are given our team's originals. The triage process is encouraging. George's lifting the exponent marathon will take a bit of decoding but lacks obvious gaps and the same goes for Alevtina's and Liam's geometry solutions. The other scripts will not take too long to digest and everything has been written up pretty clearly. A fair amount of emphasis is placed on good writing habits during training for these competitions, and it is good to see this paying off. We decide that there is time to join the team for a celebratory meal out and leave the careful marking for the following morning. This would have been completely impossible without Dominic Y, not least because brushing up on the aspects of the Miquel configuration which seem 'well known' to Liam would have taken me more time than I care to admit. The dinner includes plenty of traditional Moldovan fare, and I eat most of a rabbit. It is all very splendid.

**Friday 3rd May**

Dominic and I meet at seven to start working in earnest on the scripts. Our coordination does not start until eleven, and we finish in time to visit another cafe on our way to the Lyceum. The coordinators have evidently worked hard on the photocopied scripts, and are well prepared. To my acute embarrassment I have missed a minor gap in Alevtina's solution to Q2 which has not got past them. We hastily accept their suggested score and move on. Geometry goes well: there is no doubt that the team's solutions are basically sound, but some have been a little less fastidious with the details than they might have been. One coordinator's superb English is evinced by idiomatic use of the word 'quibble' – fortunately the quibbles do not translate into lost marks.

Further reflection on Alevtina's algebra suggests that the coordinators' initial score may have been a little harsh. Since we have had our slot already, there is no particular reason why they should find time to speak to us again, but try we must. Things are running late, and the coordinators' short lunch break is already under considerable pressure, but Galina Rusu very graciously agrees to stay behind and reopen discussion of the script. She is sympathetic, and the score is duly adjusted.

We do not expect any marks on Q4, so our last real meeting is for Q1. The creativity of George's solution is commended by the coordinators and there is no controversy until we get to Patrick's. His argument begins by invoking Mihăilescu's theorem and the coordinators are unsure whether or not to allow it. They say that other scripts like this have been awarded the non-standard mark of 8*, and that the decision as to whether or not to convert stars into pairs of points will be made by the Jury that evening. We accept the precedent and gladly finish ahead of schedule.

I have time to sample one more cafe before joining the team for pizza ahead of the final Jury meeting. It transpires that the arrangement with the starred scripts has not been communicated to Valeriu Guţu, the chair of the Jury. This leaves him in an unenviable position which is not of his making. The ensuing discussion takes some time, and the Jury are asked to vote on whether or not the theorem is 'well known'[3]. In the end the vote to convert each star into two points is carried overwhelmingly, and the rest of the meeting proceeds smoothly. Olympiad regulations state that at most two thirds of contestants from member countries should receive medals and that the ratio Gold : Silver : Bronze should be *approximately* 1 : 2 : 3. We are lucky that this year the distribution of marks makes applying these regulations straightforward: the first suggested boundaries give a ratio of 7 : 14 : 22 and nobody is inclined to argue.

**Saturday 4th May**

Today we bid farewell to Dominic Y before joining the excursion for leaders and contestants to a monastery carved into the side of a cliff at Orhei Vechi. The monastery and church are beautiful, and we also get a chance to see a traditional peasant house which has been preserved as a museum in the village. The weather is balmy, and the whole atmosphere is relaxed and cheerful. There is plenty of time to chat about life in Moldova, learn to count in Romanian and listen to a veritable orchestra of amphibian life by the riverbank.

We return to Chişinău for a late lunch and some free time before the closing ceremony. The event begins with the Moldovan national anthem, performed by four dashing young men in black tie who clearly enjoy celebrity status in the region. This group provides musical interludes between the various speeches and medal presentations.

The choreography of the ceremony gives enough time for the medal winners to wave flags and pose for photos, while keeping the event moving at a sensible pace. The presentation of Gold medals comes between rousing renditions of 'Simply the Best' and 'We are the Champions', and the two perfect scores from Serbia are appropriately lauded. The Romanian leader announces the dates of next year's Balkan Mathematical Olympiad in Romania, and promises that Preda Mihăilescu will put in an appearance to talk about the Catalan conjecture.

The ceremony finishes slightly early, and we move on to the farewell dinner. The Olympiad has been sponsored by no less than three major Moldovan wineries, and the leaders are shepherded to a separate table, partly to enjoy a glass of the produce. I sit next to Arslan Hojiyev from Turkmenistan and learn that he once spent a semester in Southampton. We discuss the charms of Hampshire before sneaking off to join our respective teams for dessert.

After dinner the students move outside to release sky lanterns and to dance. I have explained that it is customary for the UK team to participate in any local folk dancing at these events, so I am disconcerted when the first few songs are modern rather than folk. Fortunately the dancing area remains stubbornly empty until one of our students has a word with the DJ. Minutes later scores of teenagers are merrily folk dancing in a large approximate circle, and the tone of the evening is set. The attitude of the UK team remains pleasingly Gung-ho as folk songs from a number of different Balkan countries are played. The Saudis are also keen to get involved. It seems that their national dress was not designed with this particular style of dance in mind, and one or two keffiyehs fall by the wayside. A great time is had by all.

**Sunday 5th May**

Departure day seems to have come round very fast, though we agree on reflection that a huge amount has happened since we first met at Stansted. There is time for breakfast in one last cafe before we leave, and our guide is kind enough to come with us to the airport. Security is painless, and the team pass the time discussing one of my favourite problems[4].

The flight is uneventful, and all too soon the team has dispersed at the end of an unforgettable week.

[3] It is.

[4] For which $k$ can queens be placed on an infinite chessboard such that each is attacked by exactly $k$ other queens?