## A Report on the 2014 Balkan Mathematical Olympiad

By UNK7, a Ghost Of A Team

## Introduction

Guess what? This is the student report for the 2014 Balkan Mathematical Olympiad, held in the one and only Pleven, Bulgaria. If you just wish to find out how much we scored, rather than any actual details of the trip, see below. The medal boundaries were 19 for Bronze, 33 for Silver and the top score, 40, for Gold. The UK came 10th overall, out of 20 countries.

| Contestant | Q1 | Q2 | Q3 | Q4 | Total | Result |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| UNK1 Joe | 10 | 2 | 10 | 10 | 32 | Bronze |
| UNK2 Liam | 0 | 10 | 0 | 0 | 10 | Honourable Mention |
| UNK3 Neel | 0 | 10 | 0 | 3 | 13 | Honourable Mention |
| UNK4 Linden | 10 | 10 | 10 | 4 | 34 | Silver |
| UNK5 Kasia | 0 | 10 | 0 | 4 | 14 | Honourable Mention |
| UNK6 Harvey | 10 | 10 | 0 | 10 | 30 | Bronze |
| UNK7 GOAT | 2048 | 65535 | e | 3 ^^^ <br> 3 | 42 | Disqualified for being a goast |
| UNK8 Min | 0 | 2 | 0 | 0 | 2 | This is why he doesn't exist |
| UNK9 Max | 10 | 10 | 10 | 10 | 40 | Gold (but only just) |

Liam received the most notable score, in that it was the minimal score required for an honourable mention. Joe was one mark below the silver medal boundary, and so was awarded what we have dubbed a negative silver, despite its somewhat bronzelike hue. A copy of our paper can be found here http://bmo2014.eu/ps/BOM_english.pdf
The events in this report are as close to chronological order as we could make them, and any inaccuracies are entirely the fault of a space-time anomaly located in Pleven, rather than any mistakes on the part of the writers. Or our failure to take any form of notes during the event itself, deciding instead to reconstruct events from memory. If any actions or statements are attributed to the wrong person, we blame our maths-induced agnosia.

## Day 1

We arrived at the hotel one by one. No one, including Jack, seemed to know where Liam was, so Joe and Kasia went to look for him, locking Harvey out in the process. We also locate the
stairs in case the lifts turn into flaming cages of doom and destruction. Jack turns out to be the only one not to have eaten so leaves us to get food. Kasia asks for a glass of water and very nearly gets a plate of lettuce FORESHADOWING (not sure how that was misheard...). In the lobby, the four of us discuss how Pleven will most likely turn out to be a town with two men and a goat, but without the goat EPIC FORESHADOWING. Neel misses his stop and gets lost somewhere in Slough, but eventually finds his way to the Holiday Inn. Having finally gathered all except Linden, we invaded the kids area and played Lego Star Wars. Harvey went slightly crazy and chopped up everything to gather studs, while Neel disapproved of his killing of nameless henchmen for no good reason, and instead advocated the conversion of them to our side through the power of love and friendship. Sadly we were unable to find the hug combo. Conversation then turned to why exactly anyone would apply to become a nameless minion given the job's extremely high mortality rate. No clear consensus was reached. Eventually an actual kid arrived and we felt obligated to leave, so we went to bed. Linden eventually arrived at some point before midnight.

## Day 2

Horrendously early, we were woken up. After waiting for Linden to get down, we left for Heathrow. Waiting for Linden begins to become a trend. At the airport, we had breakfast and watched Johnny English while Neel attempted to mentally do a past geometry question. This was less because of a desire for intellectual challenge, and more to do with the fact that he was initially too lazy to draw a diagram. We eventually solved it by a trig bash. We also watched Jeremy Clarkson say a very bad word (on mute) - this was confirmed by a later free newspaper. Neel's passport looked suspiciously unlike him, but this turned out not to be an issue. More problematically, we lost Linden due to his foreign passport. The flight was fine, even including a second breakfast, and much maths was done. Slightly disappointingly, there was no lunch waiting for us in Sofia, only a minibus containing a small portion of the Bulgarian team. On the $2+1 / 2$ hour trip to Nnebeh (Pleven) we tried to sustain a conversation but it soon descended into a mess of fairly nonsensical bad puns involving the word "scapegoat". E.g. What do you call a goat that rolls around? A skategoat. Why are there no goats in Pleven? Because Pleven ate goat. Eventually we goat bored, having plumbed the depths of words ending in ate, ape or ake. We arrived at the hotel and met our guide Stefan. By this point, we were fairly hungry, so after a modicum of unpacking, he took us to the excitement of a Bulgarian supermarket, where, feeling pretty foreign, we stocked up on rice cakes and biscuits. Back at the hotel we wait around for a while and then have our first dinner. As it turns out the Bulgarians have salad with everything. This salad was always cucumbers, tomato, and exactly one black olive, and everything was always grilled chicken (or as the menu had it, chiken) and boiled potatoes. After dinner, trying to get upstairs to unpack, we are introduced to the magical AI of the lift system. It appears as though the direction the lift moves is determined by a Geiger counter and a sample of uranium, as it is impossible to predict where the lift will go and there is even a button in the lift with the symbol for radioactivity. We are also intrigued by the button to the mysterious floor 14, which was currently off but appears to lead to a grand panoramic restaurant. FORESHADOWING We try to find the stairs for an alternative to the "flaming death cages of doom and destruction", but since we find access to the stairs on every floor except the ground, we resort to lifts for today. It turns out we forgoat to bring flags for the ceremonies.

## Day 3

This day is the day of the opening ceremony. After waiting for Neel and Linden, we have an authentic Bulgarian breakfast, then we goato the venue - a park with cannons. The band are extremely keen and have the world's most enthusiastic piccolo player. We sit in the sun and listen for what seems like hours, followed by an equal length of speeches. Eventually, we are called to stand team by team, and when it is our time we display our handmade paper flags of the UK and Wales. We are surprised by the lack of folk dancing, given the dire warnings we had received beforehand.


After this, we seize the opportunity to visit the museum - it does feature sardines and swans, after all. Meanwhile, Neel and Joe attempt to mentally do inversion problems, with surprising amounts of success. However, two rooms of stuffed animals doesn't occupy us for long, even with the many exciting buttons to press, so we amble back to the hotel in time for lunch.
We were attempting to be sociable by sitting in the only obvious common area of the hotel, but no other teams share this mentality, so we spend the afternoon mathsing and trying to learn how to play Bulgarian card games from the guides.
It was at this time that we discovered Joe's closet Satanism, through his compulsive drawing of pentagrams. However, despite this incontrovertible proof of his beliefs he still denies it. Perhaps he has not been convinced that our team would still accept him if he was more open about his faith. Harvey then shows us how to construct a pentagram, which leads him to look up how to construct larger shapes, culminating in him attempting to constructing a 17-gon. (See Appendix A)

For unknown reasons, we then play large amounts of 2048, and decide to give up on winning in favour of trying to lose as fast as possible. Harvey manages to do so with a score of 0 , using methods which are totally not in any way cheating. Ahem.

We are up till fairly late playing Mao - perhaps before an exam this wasn't the best of ideas, but Neel's rule is only guessed by Joe, contrary to his claim of it being really simple, and we couldn't leave it at that. Harvey - for which of course we are all grateful - tries to teach us the Welsh national anthem by force, and any pain our mispronunciations caused him was entirely his fault. (See Appendix B.)
This leads to Neel discovering that the English national anthem is far more interesting than just the first verse of "We are both Christian and royalist", extending to "The Queen is great and we should give her free stuff", "colonialism is awesome" and "God help us to crush the rebellious Scots".

## Day 4

We did some maths in the morning, apparently. This probably wasn't a significant enough event to waste words talking about. Not much notable occurs, although the confused face Kasia pulls when the instructions are read out causes the invigilator to ask if she understands English. This invigilator later steals her orange highlighter.
Okay, a short moan is possibly in order about the paper. Who thought Question 2 was a good idea? And why oh why was Question 4 so easy to misunderstand? Would it truly have been so difficult to put in a clarifying note? Around half the competition made the same mistake of assuming that the sides of smaller hexagons were parallel to the sides of the larger one. And why did Question 1 have such an obvious-in-hindsight trivial solution? And why do geometry problems exist and have such pretty solutions?
Upon getting back to the hotel, we discovered that with the paper done, other teams were now willing to socialise, and we spoke to the Albanian team. They are surprised that despite his stature, Joe is the third youngest member of our team, being older than both Neel and Harvey. The non-perfect squares on our team, are mortified to discover that despite our discussion of power of a point before the exam, we did not spot the solution involving the radical axis theorem to Question 3. We can only hope that Geoff finds it in his heart to forgive us.
Maybe we were feeling dejected, maybe it was the rain, but we do not leave the hotel all afternoon. We introduce Mao to the Bulgarians, who eventually all give up in disgust as their confusion between spades and clubs is exacerbated by Harvey's new rule, where the even spades and clubs are switched. We move on to Mafia, which goes down better, and we are taught Logic Mafia, a 5-player variant where there is 1 mafioso and 4 detectives, sane, crazy, paranoid and naive. The sane detective always gets the right answer, the crazy is always wrong, the naive always gets innocent and the paranoid always gets guilty (See Appendix $C$ for Harvey's suggested extensions). Jack, released from his holding cell on the other side of Pleven, comes to join in. This makes it dinner time.
Over dinner, Neel introduces us to various thought experiments such as Newcomb's Problem and Pascal's Mugging, which confuse the hell out of us but at least make for interesting conversation. If you have not heard of these before, we recommend googling them. They're fascinating problems.
Jack raises the possibility that Joe's compulsive drawing of pentagrams could be caused by demonic possession, a viewpoint we had not considered. We then debate whether or not Joe's ability at maths comes from possession by a particularly mathematically inclined demon, or if it is his own natural talent which the demon is exploiting. During this, Joe has his head buried in his hands, something we can only attribute to some form of possession induced headache.

In a completely unrelated series of events, Linden gets adopted by the Albanians. After dinner, half the UNK team discover that they have lost their keys. Thankfully, someone handed them in at reception. Neel subsequently insists on having custody of their room key, lest Linden lose it again, given his track record of $100 \%$ failure.

## Day 5

Neel and Linden are late for breakfast. Again.
Whilst Jack and Gerry are in coordination, we are taken to see the panorama. It is pretty amazing, a kind of sculpture painting combo that makes your eyes water trying to see where they join and scenes which simultaneously look very far away while being a short distance from your eyes. This leads to several of us getting headaches. Apparently, it's of a very important battle in the history of Pleven, but Linden and Neel have been recently introduced to Nim, so are too busy scribbling theories to take in the view. Meanwhile, a guide disapproves of our constantly doing maths. We are still unsure why anyone would object to this. When we emerge, the buses have deserted us, so we are taken through the park to a statue that Harvey declares a giant phallic object. Luckily its true name, Mother Bulgaria, is picked up by some of the rest of the team and stored for later. FORESHADOWING


It begins to rain, to the disappointment of everyone except Neel, who is surprised to discover that despite being British, the rest of the team are not fans of rain. They are forced to huddle under Kasia's umbrella, while he enjoys freedom of movement.
The scores are revealed gradually around this time, and we find out that mentioning lettuce in question 4 was worth an extra mark. Neel's induction solution, sadly, was not worth this mark as it contained no vegetation. Liam has received no marks for this question, due to his tragic ability to correctly interpret questions. Much speculation is begun as to the medal boundaries. Significantly more people appear to have goatten 40/40 than normal, which does wonders for many of our team members' self esteem. Our suspicions are later proven correct, when the medal boundary for gold is 40 . Neel is informed by Jack that there is not a non-zero chance that the bronze medal boundary will be $\leq 13$.
Most of our team now discovers that in many olympiads, if you can solve a problem by assuming a crucial step which does not trivialise the solution, you will get partial marks. We wonder why no one told us this before, and decide to notify next year's Balkans team. In the afternoon, the guides organise a great race for the more energetic teams. Somehow, this includes us, even though Kasia's shoes repeatedly fall off, such that at one point she is running down a road barefoot. Impressively, we are the second team to the second checkpoint, the aforementioned statue, due to Joe's memory of Pleven's geography. From here we rely on Linden's amazing ability to ask for directions in Bulgarian as no one else is willing to speak to strangers, aptly aided by Stefan doing anything except guiding. Although we are under the mistaken impression that we are supposed to find Bulgaria Square for most of the task, and it starts raining heavily with only two umbrellas between us, we manage to get to Macedonia Square, where we are informed that we have won and collect our prizes of chocolate. Linden refined his ability to ask "Ploshtad Macedonia" in a questioning manner to a high degree, a skill we hope he shall retain for the rest of his life. Returning to the hotel, we do not rub this in the face of the Albanians at all.
Gerry now showed us various mathematical problems related to the school syllabus, some of which we later discover were reused from the previous year's Balkans. They included the problem of how many times you must flip a coin for the probability of getting all heads to be less than $1 / 10^{10}$. This is of course equivalent to $1 / 2^{n}<1 / 10^{10}$, and taking logs of both sides yields $n * \log (1 / 2)<10 * \log (1 / 10)$, or $n<10 * \log (1 / 10) / \log (1 / 2)$, an upper bound for $n$ which is clearly false. This can be explained if $\log (1 / 2)<0$, as this is dividing by a negative and so flips the inequality. But if you take $\log$ base $1 / 4, \log (1 / 2)$ is $1 / 2$ and so positive. Harvey quickly deduces that the problem now lies in the first step in a Sherlockian style as "it can't be any of the others". We then notice that log base $1 / 4$ is a decreasing function and so the inequality ought to be flipped when you took the log at the beginning.
After dinner, Neel is given a major hint with Nim by Joe and eventually solves it, although has no idea why the solution is correct.
Stefan turns up with monopoly money, and the Albanians wander over to defend their honour through the medium of poker. Initially, we play fairly leniently, allowing players to buy in for free, but the Albanians push for a fresh game with harsher rules. This leads them to a narrow victory. As we are in the mood for gambling, the Romanians arrive with a mathematical bet - can we solve their problem in 15 minutes? Answer: no. Even having given it to Gerry and Jack it remains unsolved by the end of the competition. If anyone is able to find the apparently existent 1-line solution, please let us know.

Given a triangle $A B C$ and its circumcircle, draw in a chord parallel to $B C$ in the same segment as $A$. Construct the two circles tangent to this chord, the circumcircle and $A B$ and $A C$ respectively, outside the triangle. Prove that the line between the points of tangency with $A B$ and $A C$ is parallel to $B C$.
It gets later and later, and the smaller UNKs have headed to bed. Kasia begins acting very oddly (read: the stage of exhaustion that resembles drunkenness) and almost collapses onto Liam, who is unsure that her ability to quote R.E.M. lyrics compensates for this. Everyone except Linden makes the sensible decision to get some sleep at this point.

## Day 6

Undoubtedly the highlight of the entire trip was the two hour coach journey to visit a ruined castle. Admittedly, the journey was livened up by Contact and Cucumber (arbitrarily many questions guessing a verb), but never has climbing up and down a small hill several hundred kilometres from our hotel been so thrilling. The day goat off to a good start, with a series of extremely creepy plastic statues by the entrance, but fortunately the tourist trap aspects were soon left behind us. To add some degree of interest, Harvey and Neel kept up a Pokémon based text adventure game for over an hour, which somehow moved from a wild castle appearing to Harvey being eternally tortured in hellfire for eternity until he woke up again at the entrance to hell. Rest assured that he eventually found his way back to the material dimension, where he was forced to fight his way through 666 animated Satanic altars. His attempts to use Pokémon attacks were somewhat stymied by Neel never having played Pokémon. Many were surprisingly ineffective.


Neel wishes to make it clear that his expression is due to his aversion to bright sunlight

After the joy that was the castle, the next stop on our sightseeing tour was a waxwork museum on the history of the town, the most realistic model in which walked off as Kasia admired it. Of course, the statues were so captivating that we spent most of our time playing various computer games on the touchscreen computers that were supposed to be playing clips of Bulgarian history and culture. Having experienced all that the town could give, we tried to contain our joy in the coach back.
Mid-journey, we receive more questions about Joe's age. People continue to be surprised to discover that he is in fact the third youngest member of our team. We blame the widespread use of the heuristic that height correlates with age.
Upon getting back to the hotel, Neel, Joe, Linden and Harvey decide to explore the mysterious Floor 14, which is currently open. Upon reaching it, an angry looking Bulgarian man waves us back towards the lift and we leave. The results of our fact-finding mission were inconclusive. To top off the day was the closing ceremony, the most glorious butchery of the English language (even noted by the Bulgarian team) which we had ever had the pleasure of experiencing. The presenter called it an award-winning ceremony, and by the end we could definitely see why.
It began with folk dancing. This continued for close to an hour, accompanied by music about 2 octaves too high and 20 decibels too loud. We were sitting close to the back, as befits our place in the alphabet, so we can only imagine the torment undergone by those closer to the front.


Next, the speeches. Really, a special mention to whoever thought George Orwell's "Animal Tyrannical" was a good choice of source material. It did have the entire team in hysterics. "All animals are equal, but some are more equal than others." -Please stop- "If we apply this to humans..." -Not really a good idea- "... all contestants are equal for having participated in the same competition, but those who scored higher are more equal than others." This fine lady then apologised, but not for this interesting sentiment (See Appendix D for followup). Other quotes we are happy to attribute to her include "The show must go on, get off the stage." and "The stage is a lovely place, and I would like to invite the mayor of Pleven to it - I mean the stage.". Linden, Joe and Harvey were all invited to the stage to receive their medals with our minute hand-drawn flags, in order to represent the UK's artistic spirit and creativity, in contrast to the unoriginality of the other teams and their more conventional flags. Credit for the Welsh flag goes to Joe, who put his majestic GCSE art skills to good use. Sadly we were subsequently oneupped in terms of size by Serbia with a flag they had stolen from the jury meetings, with Serbia
conveniently typed beneath. Our current plan for future events is to get a picture of a flag on one of our phones, or to construct one from carbon atoms.


One of the sponsors decide to give the Bulgarian team a smartphone each. Sadly, the Bulgarian team does not accept applicants from the UK. Speeches and medals done, we goat in as many teams' photos as possible and went off to the final dinner.
On the way back to the hotel, Liam's Corollary was developed "All numbers are equal, but some are more equal than others". This has as a trivial consequence that " $2+2=5$ ".
The final dinner was much fancier than the previous dinners, complete with toothpicks, complimentary wine and a DJ. It featured not only chiken but becon, too. Bored of a Careless Whisper and Lady in Red style soundtrack, the UK initiated a takeover of the sound system beginning with the Macarena but, as other teams joined in, covering everything from folk dancing to the penguin dance, from Gangnam Style to the Numa Numa. This led to Neel
discovering a hitherto unknown passion for dancing, and his subsequent disappointment with the rest of the team for being somewhat less enthusiastic.


When the Bulgarian singing started, everyone but Linden took this as our cue to leave, and given that it was gone midnight and we hadn't packed yet, this was probably necessary.

## Day 7

On the morning of the last day, Neel awoke to find Linden sprawled insensate upon his bed. After enduring 3 of Neel's attempts to wake him and sleeping through his own alarm, Neel was forced to leave his phone alarm on next to Linden's head for a few minutes before he finally managed to rouse himself enough to make it to breakfast. Surprisingly, they were only 5 minutes late. Upon finishing, Linden and Joe decided that the most logical course of action would be to sprint up the stairs to their room, as the lifts were even less reliable than our sense of goat humour. Upon reaching his destination, Linden collapsed in exhaustion on the corridor floor, whereupon he was discovered by the hotel staff. Suspecting that he had suffered a medical emergency, an ambulance was called, as was relayed to us in broken English and mime, which we had initially taken to mean "tennis ball". Thankfully, after a few medical tests, he was released with a clean bill of health. However, these tests result in him being late for the coach back to Sofia. In this case though, we concede that he is not entirely at fault.


On the coach ride back, Harvey introduces Neel to Fibonacci 2048, a far more confusing variant. Neel then decides that the most efficient use of his remaining Levs is to buy an 11 Lev bar of Dairy Milk. We also decided to have lunch in the SkyCafe where most of us order a pizza baguette. Our waiter informs us that there are no more ham pizzas left and that he would order pepperoni for everyone. He tells us to indicate if we specifically do not wish to have pepperoni, or else we would be in a perpetual state of having the pizza in our hands but not eating it. Or rather, as Linden put it, "Speak now or forever hold your pizza". We reach the plane with no further incident.
On the flight back Neel solved an IMO inequalities problem with Lagrangian multipliers, an occurrence notable for his consistent use of maths. He is then looked down upon for the hideousness of his methods. Despite Lagrangian multipliers being used in one of the official solutions, he is unable to convince anyone else that it is a legitimate method to do maths with. At Heathrow Linden discovers via text that in light of his silver medal, he has been invited to Oundle, something we all agree was well-deserved. He is then held back at passport control, making us wait for him at baggage reclaim, thanks to his Australian passport. His mother brought chocolate for the entire team, a well-appreciated gesture.
A most interesting and unique event happened on Neel's journey home from Heathrow, which this report is too short to contain.

## Conclusion

We all thoroughly enjoyed the competition and our stay in Pleven. Our thanks go to Jack and Gerry for taking care of us while we were in Bulgaria and for marking our scripts. To our guide Stefan, for being a wonderful and friendly addition to our team, keeping us entertained and preventing us from getting hopelessly lost in Pleven. To UKMT, for organising and funding everything. To Bulgaria, for hosting the event. To the atmosphere, for preventing our asphyxiation. And finally, to the laws of physics, for our continued existence.

## Appendix A: Harvey's ultimate construction of the 17-gon:

To see the true beauty of this construction first we must see how it was arrived at algebraically. First we must look at the seventeenth root of unity, omega, being a solution to the equation $x^{16}+x^{15}+x^{14}+x^{13}+x^{12}+x^{11}+x^{10}+x^{9}+x^{8}+x^{7}+x^{6}+x^{5}+x^{4}+x^{3}+x^{2}+x^{1}+1=0$. Obviously there are 16 solutions to this equation, being the first to sixteenth powers of the seventeenth root of unity.
Next we construct two sets:
$\left(\omega^{1}, \omega^{2}, \omega^{4}, \omega^{8}, \omega^{16}, \omega^{15}, \omega^{13}, \omega^{9}\right)$
$\left(\omega^{3}, \omega^{6}, \omega^{12}, \omega^{7}, \omega^{14}, \omega^{11}, \omega^{5}, \omega^{10}\right)$
Note that these are gotten by repeatedly squaring the first term. Next we find the two sums of these sets' terms.

$$
\begin{aligned}
& s_{1}=\omega^{1}+\ldots+\omega^{9} \\
& s_{2}=\omega^{3}+\ldots+\omega^{10}
\end{aligned}
$$

Now when we add these two terms we obviously get $\omega^{1}+\ldots+\omega^{16}$, which is -1 .
But when we multiply them we actually get exactly 4 times the sum, or -4 . We can now solve the quadratic to get $s_{1}$ and $s_{2}$ :
$s=(-1 \pm \sqrt{17}) / 2$
To figure out which one is $s_{1}$ and which is $s_{2}$ simply note that $s_{1}>s_{2}$.
Next we find $t_{1,} t_{2}, t_{3}, t_{4}$ by finding sums of repeatedly taking fourth powers.
$t_{1}=\omega^{1}+\omega^{4}+\omega^{16}+\omega^{13}$
$t_{2}=\omega^{2}+\omega^{8}+\omega^{15}+\omega^{9}$
$t_{3}=\omega^{3}+\omega^{12}+\omega^{14}+\omega^{5}$
$t_{4}=\omega^{6}+\omega^{7}+\omega^{11}+\omega^{10}$
Then
$t_{1}+t_{2}=s_{1}$
$t_{1} t_{2}=-1$
$t_{3}+t_{4}=s_{2}$
$t_{3} t_{4}=-1$
So $t_{1}, t_{2}=\left(-1 \pm \sqrt{s_{1}^{2}+4}\right) / 2$
and $t_{3}, t_{4}=\left(-1 \pm \sqrt{s_{2}^{2}+4}\right) / 2$

Now finally we let $u_{1}$ to $u_{8}$ be the sums of the conjugate pairs of the powers. We then get four pairs of simultaneous equations just like the ones above, but with different sums and products.

It is left to the reader to finish the final stages of the algebra.
Now for the construction:
Draw a circle with radius 2 and centre A. Draw two perpendicular diameters, and label one of the endpoints $F$. Next on the other diameter mark off point $B$ distance 0.5 from $A$, then draw a circle with centre B through F. This circle meets the line $A B$ at $G$ and $H$, where $G$ is further from A than H. Now draw FG and FH, and the three circles with centres F, G, H and passing through A. The intersections of the circle with centre G and FG are $T_{3}$ and $T_{4}$ where $T_{4}$ is further from F than $T_{3}$. The intersections of the circle with centre H and FH are $T_{1}$ and $T_{2}$ where $T_{1}$ is further from F than $T_{2}$.

The final steps are left as an exercise for the reader. They involve constructing $2 u_{1}$ to $2 u_{8}$. To construct $2 \sqrt{t}$ you draw a semicircle with diameter $2 t+2$, and see where the perpendicular goes. After that, the key idea is to use Pythagoras.

## Appendix B: The Welsh National Anthem:

Mae hen wlad fy nhadau yn annwyl i mi
Gwlad beirdd a chantorion enwogion o fri
Ei gwrol rhyfelwr gwladgarwyr tra mad
Dros ryddid collasant eu gwaed
Gwlad!!! Gwlad!!! Pleidiol wyf i'm gwlad
Tra mor yn fur i'r bur hoff bau
O bydded i'r hen iaith barhau

## Appendix C: Logic Mafia Extended:

A) There are 10 players. 2 are mafia, 1 normal mafioso and one Godfather. There are the 4 regular detectives from original Logic Mafia, for whom the Godfather registers as the same as an innocent villager. And there are another 4 detectives, who get the opposite result for the Godfather and the same result for everyone else as their counterparts do.
B) There are 19 players: 16 detectives, 1 mafioso, 1 godfather, and 1 miller. The miller is simply a normal citizen who doesn't do anything during the night. Same as above, except the extra 8 detectives find the miller as the opposite result as their counterparts.
C) There are 35 players. It is the same as Version B, except as well as of 1 miller we have 16 miller-detectives who all have a normal detective counterpart. A normal detective always gets innocent for a miller detective and a miller detective always gets guilty for a miller detective
D) There are 133 players: Two mafia groups, each with one normal mafioso and one godfather. There are 64 normal detectives and 64 miller detectives, and 1 miller. Each of the 64 possible innocent-guilty possibilities is for one normal detective and one miller detective.
E) Generalised Logic Mafia, for an arbitrarily large number of participants. There are $2^{n}+n-1$ players. There are $n-1$ mafia members, numbered from 1 to $n-1$. There is a bijection between detectives and binary words of length $n$, where the value of the $k$ th letter is the value the detective gets for that mafia member ( 0 is innocent, 1 is guilty) and if $k=n$ that corresponds to the value that detective gets for each regular detective
F) Cantor's Impossible Logic Mafia: Each detective is numbered $1,2,3, \ldots$ and there is one mafioso who is numbered 0 . There is a bijection between the detectives and binary numbers of length $n$, where $n$ is the number of players, and where the value of the $2^{k}$ th value place is the value the detective gets for the $k$ th person. Naturally this is an impossible game if the number of people playing is more than 2, even if the number of players is infinite because Cantor proved that $2^{n}$ is always greater than $n$.

## Appendix D

Since our return to the UK, our guide Stefan informed the presenter at the closing ceremony of our thoughts on her creative use of that Orwell quote. She replied that "she was allowed to apply it in a different context" and "that we should be more broad-minded". From this we can only conclude that she has not actually read Animal Farm, as that was said with a straight face..

