UK IMO team leader's report

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After IMO 2001 in Washington, I took over as UK leader from my eponymous predecessor Imre of that ilk. I had been recruited over lunch at Trinity College Cambridge by Adam McBride. His references to "willingness to serve" initially led me to believe that he was either about to offer me a job on the college staff, or to suggest a game of tennis. Naturally my first thoughts concerned who had already turned the job down, but Adam gallantly assured me that no-one had done so. It seemed unlikely that this was an outright lie, but it all seemed very odd. Then the truth struck me. Everyone with the appropriate experience and talent was tied up running the forthcoming IMO in Glasgow, so Adam was being forced not so much to scrape the barrel as to gouge it.

It so happened that I had already planned a trip to Budapest in the aftermath of the Washington IMO. This took on a new significance, and thanks to the kind help of Richard Rimanyi, a Hungarian singularity theorist I had met through a conversation in a Turkish bath, I was introduced to the Hungarian IMO establishment. Happily I already knew their leader Jozsef Pelikan because of our mutual interest in group theory. Sandor Dobos is the Hungarian deputy leader and chief trainer, and he proved very helpful. Sandor and I started to develop plans for joint training under the auspices of the Bolyai Society.

Upon returning to the UK, these plans were refined with the help of our splendid deputy leader Richard Atkins. We received generous backing from the BMOS and UKMT for a much expanded training programme. Richard and other sages, gurus, veterans and relics of the British Mathematical Olympiad scene helped me to draw up a list of likely prospects for the UK IMO team in 2002. These students gathered for our first training camp at the University of Bath in September 2001. We did a great deal of mathematics, and the students started the process of bonding into a squad. We had sessions from many experienced trainers, and UKMT supremo Peter Neumann also paid a visit. The attacks on the United States on September 11th happened during this camp, and the mood quietened as we reflected on the New Yorker who had been the UK's guide in Washington, and everyone else caught up in the trauma. Happily the guide and his family were safe, but one of the senior executives of the Akamai foundation (one of the main sponsors of IMO Washington) had been killed.

During the Autumn of 2001 the advanced mentoring scheme kicked in for the new squad of 15 students, and we decided to have monthly exams. At this stage Tim Austin, Nathan Bowler and Paul Jefferys were the leading candidates for places in the team, but the remaining three places were wide open.

After Christmas we gathered at Heathrow and travelled to Budapest. Our Hungarian friends had organized a wonderful mathematical programme. We stayed in a boarding school in conditions which were perfectly comfortable but came as something of a surprise to the less well-travelled students. In particular, the collective shower rooms were a novelty. Of course, Richard and I had grown up in an era when the treatment of

teenagers was more robust than today. The girls took action and posted a rota and warning sign on the door of their shower room. The male reaction was muted, and the more sensitive souls took to showering in the middle of the night (if at all).

A research student from the University of Bath, Ceri Fiddes, was there to provide adult support to the four girls who came on the trip. Every morning we had lectures from Hungarian academics, and in the afternoons we worked on problems with the Hungarian squad. In the late afternoon the students presented their solutions, and in the evening the UK squad sought to undermine the future of Hungarian mathematics by teaching a pointless but addictive card game called Mao. New Year celebrations in Budapest were a great success. Tim Austin's birthday was celebrated by the presentation of a remarkable polychromatic jester's hat, replete with fronds, bells and so on.

Jozsef Pelikan gave a tour de force lecture on algebraic number theory from the ground up. His lecture (without notes) started at 9:00 am and finished at 1pm. He kindly agreed to our request for a tea break of 10 minutes in the middle. It was an exhilarating experience, and many of the students were on the edge of their seats.

We moved residence to a hotel in a suburb of Budapest called God. Over the road was another suburb called Alsogod. Thankfully inspiration (divine or otherwise) was at hand when our students sat the third round of the Hungarian Mathematical Olympiad, and they did quite well. The exam room was an airy wood panelled room with a glorious view over a garden to the Danube. In the evening there were snowball fights, and the impudent Paul Jefferys had the cheek to throw one at me. The Hungarian trip was a delight, and we must thank the Hungarian students and their brilliant trainer, Sandor Dobos.

As we returned to the UK, I went on sabbatical to the Institut Henri Poincare in Paris, but returned to London for a week-end to help with marking the first round of the British Mathematical Olympiad. We expected that the students in the UK squad would perform very well in BMO1 because of their extra training, and indeed this happened. Richard and I were watching carefully for evidence that we had missed some strong students, and after BMO2 (marked in Paris) we added some new names to the UK IMO squad for the Trinity Camp at Easter. The traditional FST (final selection test) was renamed the first selection test. As a result of the FST, both Martin Orr and Tim Northover were added to the squad, and some other students were rested.

After Trinity the training regime became more intensive, with a problem sheet once every 10 days. At the start of June we had a camp at Oundle School over the Golden Jubilee week-end. At this stage we were joined by a film company who had been commissioned by BBC2 to make a documentary including our preparations for (and participation in) the Glasgow IMO. The director is Stephen White, and we understand that the programme will be shown in 2003. This provided an unusual backdrop to the preparations for the IMO. After Oundle we made the final selection of the team shown in the marks table below. Bryn Garrod, Tim Northover and Martin Orr were standing by as very capable reserves.

The final run-up to IMO 2002 consisted of a week in Birmingham in parallel with the National Mathematics Summer School, and then a week at Trinity College Cambridge. By now the team were taking a mock IMO examination every day. The journey between Birmingham and Cambridge on the UK IMO bus was enlivened by Stephen's film crew buzzing around on the motorway, taking footage for the documentary.

In Cambridge we continued the preparation. Diversions included a punting trip and a visit to the Wren library to view Hardy-Ramanujan correspondence, the original Winnie-the-Pooh manuscript and other similar items.

The IMO

The leaders' hotel was in Dunblane, temporarily renamed Brigadoon. A house near the hotel carried a small plaque on which it was explained that Charles Edward Stewart has spent the night at that residence on September 11th 1745. The failure of the Jacobites to secure the Catholic succession was reflected in the conduct of the 43rd IMO. The first jury meeting was chaired by John Knox, who had risen from the dead and donned an Adam McBride mask for this purpose. He peered into our souls, and was not impressed.

The leaders were told about their duties and responsibilities in considerable detail. Telephone communication with the outside world was cut off and the Black Watch had the hotel surrounded. The preacher grudgingly conceded that (regretfully) we were in a democracy, and leaders could swan off if they wished, but then he reminded us of our consciences, and the indelible stain that we would carry if we neglected our obligations in any way. The McBride impersonator spoke more and more slowly to make sure that any non-Presbyterians present would get the point. Between invocations of wrath and damnation, at first between sentences, and later between words, time seemed to stop. He used these gaps to demonstrate exquisite control over his features, eyeballs throbbing in their sockets until inevitably they began to revolve in opposite directions.

Peter Neumann was allowed to make a few welcoming remarks by way of intermission, before the chairman reasserted his authority and continued to enumerate a rather interesting list of human weaknesses to which we should not fall prey, including sloth, tardiness and taking the problems booklet outside the building (particularly to Fort William). Messrs Bradley, Coggins, Richardson and Monk sat alongside the chairman, exuding relative bonhomie, as Adam tried to persuade us that they were the four horsemen of the apocalypse.

We had been given 27 questions by the problems group, and we had to try them all. I settled down to the task. The problems are divided into Algebra, Combinatorics, Geometry and Number Theory, and ranked in order of estimated difficulty. I bravely adopted the horizontal approach, and tackled each question 1 in turn. I impressed myself by knocking them over fairly easily, and slept soundly. After breakfast of haggis (a food substitute) I settled down to try the question 2's. This is where I came a cropper, and spent a difficult morning achieving rather little, though now I did recall the talk of having some particularly easy questions for the weak students. That explained my prowess of the

previous evening. However, after lunch things started to pick up, and some more of the questions yielded.

In the evening we were given the solutions, and suddenly the questions looked less intimidating; the benefit of struggling with them under realistic conditions was clear. The debate on the selection of the problems raged for a couple of days.
We first selected the harder questions. One was a mixture of geometry and combinatorics, and the other a fusion of algebra and number theory. After that we selected a pair of relatively easy questions. I fought hard to stop the question which was eventually to become Problem 1, producing a solution only one line longer than the question in an attempt to undermine its support. This plan narrowly failed. The question was so easy that the co-ordinators made it a very technical problem in order to give it some value as a discriminator. It is one of those questions where almost any method works, so the marking scheme was quite savage. A momentary lapse would be punished by the loss of four marks. In fact this played into the hands of the well trained students, and created major problems for the less well prepared. Later one of our students was to fall victim to this trap. It cost Tim Austin a gold medal.

The other easy question seemed rather good. The middle difficulty pair was fit for the purpose, but a bit too easy in my opinion. The jury nearly chose an even more beautiful geometry question than the one they selected for Problem 2. The English Language committee produced a refined wording of the questions. As UK leader I had to present this polished wording to the jury. Of course all jury members are fluent English speakers, so the jury made a lot of subsequent revisions.

Here are the questions:

First Day

1. Let n be a positive integer. Let T be the set of points (x, y) in the plane where x and y are non-negative integers and $x + y < n$. Each point of T is coloured red or blue. If a point (x, y) is red, then so are all points (x', y') of T with both $x' \le x$ and $y' \le y$. Define an X-set to be a set of n blue points having distinct x-coordinates, and a Y-set to be a set of n blue points having distinct y-coordinates. Prove that the number of X-sets is equal to the number of Y-sets.

2. Let BC be a diameter of the circle Γ with centre O. Let A be a point on Γ such that $0° < \angle AOB < 120°$. Let D be the midpoint of the arc AB not containing C. The line through O parallel to DA meets the line AC at J. The perpendicular bisector of OA meets Γ at E and at F. Prove that J is the incentre of the triangle CEF.

3. Find all pairs of integers $m, n \geq 3$ such that there exist infinitely many positive integers a for which

$$\frac{a^m + a - 1}{a^n + a^2 - 1}$$

is an integer.

Second Day

4. Let n be an integer greater than 1. The positive divisors of n are $d_1, d_2, \ldots, d_k$ where

$$1 = d_1 < d_2 < \cdots < d_k = n.$$

Define $D = d_1 d_2 + d_2 d_3 + \cdots + d_{k-1} d_k$.

(a) Prove that $D < n^2$.

(b) Determine all n for which D is a divisor of n^2.

5. Find all functions f from the set $\mathbf{R}$ of real numbers to itself such that

$$\big(f(x) + f(z)\big)\big(f(y) + f(t)\big) = f(xy - zt) + f(xt + yz)$$

for all x, y, z, t in $\mathbf{R}$.

6. Let $\Gamma_1, \Gamma_2, \ldots, \Gamma_n$ be circles of radius 1 in the plane, where $n \geq 3$. Denote their centres by $O_1, O_2, \ldots, O_n$ respectively. Suppose that no line meets more than two of the circles. Prove that

$$\sum_{1 \leq i < j \leq n} \frac{1}{O_i O_j} \leq \frac{(n-1)\pi}{4}.$$

Problems 1 through 6 were submitted by Columbia, (South) Korea, Romania, Romania, India and Ukraine (respectively).

We were bussed to Glasgow for the opening ceremony. I saw the team but they did not see me. Naturally we were entertained by a bagpiper. More surprisingly, an Australian juggler named Colin Wright gave a dazzling performance. The leader of Poland described him as “the remarkable man with three balls”.

Next day the students had their first exam and the jury was entertained by the students’ queries. Happily for the documentary crew and public entertainment in general, one was from our very own Jenny: “Am I missing something or must all the points be red?”. I pointed out to the jury that this was a disjunction and the first clause was true, so that the correct answer to the question was yes. I craved the jury’s permission to write “Read the question again!”, and was allowed so to do. That evening the scripts were distributed quite late, and there was hardly time to look at them before falling asleep.

After the questions next morning the jury boarded coaches to meet the students at the end of the second paper. I met the students only briefly before we became separated, and then we failed to have lunch together thanks to some dim planning in the restaurant. After

being away from the team for so long I was very angry that I did not have the chance to talk to them properly. The students vanished and Richard and I were left to mark. Initially things looked quite good, but after a while it became clear that Nathan's script was a traffic accident. The scripts of Paul, Tim, Jenny and Tom all contained lots of good stuff, and there were heart-warming moments in Gavin's work.

Only Tim had made even minor progress on Problem 6, and no-one had made more than a little dent in Problem 3. However, the relatively easy problems had been done rather well by many of the UK team, and this served to validate the training effort. We had swept up lots of geometry marks from Problem 2, often using trigonometric formulas learned for this very purpose.

The Results

The dramatic moment of the co-ordination phase was when Richard and I believed (correctly) that the cut-off for gold would be 29 and that Paul Jefferys had solved the remaining two questions completely, and so would be on 29. There was a slight niggle in that one could argue that he had paid insufficient attention to a detail in a geometry proof, and might get only 6/7 for that. My heart sank when I sat down to co-ordinate that question because the South Koreans who were running the show had written their proposed scores in their own language on a pad. I could see that they were proposing to give the same squiggle to Paul as to Jenny. Since Jenny's solution was a clear 6/7 we were in trouble. When it came to the script, Richard and I made out a clear case as to why it should be 7/7. The case was strong enough for the Koreans, who nodded in agreement that 7/7 was justified. For a moment we relaxed, yet the gold was snatched away by the (British) co-ordinators, who argued that it was case law that Paul's script was worth 6/7. We acted astonished, and adjourned to paw over the script looking for further evidence. However, it was not there, and the gold medal slipped away. This was a great disappointment, but serves to illustrate the scrupulous fairness and quality of the co-ordination team.

Tim Austin was also very unlucky to miss out on a gold medal. His sin was correctly to dispose of infinitely many base cases of a structural induction. He messed up one easy case, and was fined 4 marks. There were well rehearsed arguments as to why this was just, repeated *ad nauseam*. I do not find repetition a very persuasive argument, and still think that a fine of 1 mark would have been appropriate (with the more severe penalty for those who failed to deal with infinitely many base cases). Anyone who thinks that they can counter this position by rehearsing the standard argument wheeled out in Glasgow had better seek a career in politics rather than mathematics. I really did understand first time, and hearing it again and again does not help. The only argument which has real force is that the marking scheme mugged everyone in the same way, provided that they were unlucky enough to use one of the induction arguments with a delicate base case.

Tom and Jenny were also very unlucky to fall short of the silver medal threshold. In their cases it is the sheer proximity of the silver borderline which is so annoying. Gavin has been improving all year, and to have him stumble at the IMO was a surprise. Those of

you who know Nathan will realise that he is a far stronger mathematician than these IMO marks would suggest. He is determined to harness his remarkable talent to much greater effect in the future.

The marks were as follows.

	Q1	Q2	Q3	Q4	Q5	Q6	Total	Medal
Paul Jefferys	7	6	1	7	7	0	28	Silver
Tim Austin	3	7	1	7	7	1	26	Silver
Tom Coker	7	6	0	7	2	0	22	Bronze
Jenny Gardner	6	6	0	7	2	0	21	Bronze
Gavin Johnstone	6	0	1	4	1	0	12	
Nathan Bowler	3	1	1	1	1	0	7	

A gold medal was available for 29 points, a silver for 23 points, and a bronze for 14 points. Thus 9 extra marks, correctly distributed, would have secured two gold medals, two silvers and a bronze! However, it was not to be this year. Uncharacteristically ragged performances from Gavin and Nathan had the effect of depressing our position in the national rankings. If these two had each secured 20 marks (well within their capabilities almost all the time), then the UK total would have increased from 116 to 137 and our ranking would have been 14^{th} rather than 27^{th}.

The leading teams were: China 212, Russia 204, USA 171, Bulgaria 167, Vietnam 166, (South) Korea 163, Taiwan 161, Romania 157, India 156 and Germany 144. Only three students secured full marks (42). Two of these students were from China and one was from Russia. There was then a clear gap to a group of six students on 36 points. A student from New Zealand managed the stunning feat of securing a gold medal with 29 marks despite scoring 0 on the easiest question (Problem 1).

The UK was near the bottom of a close pack of moderately successful teams. There was a clear break between the 30^{th} team (Singapore) on 112, and the 31^{st} team (Argentina) on 96. There were 17 countries in the narrow range 112 through to 135 inclusive.

The monotonic decline in UK scores and ranking since 1996 has been halted and reversed. Our medals in 2002 were all strong, towards the top of the range. Four of the team are available for selection in 2003: Nathan Bowler, Jenny Gardner, Gavin Johnstone and Paul Jefferys will all still be in secondary education next year. Of course the team selection is not yet made, and these people must compete with other students who may emerge as leading young mathematicians before IMO 2003 in Tokyo. We are all determined to make the team of 2003 a serious force.

Conclusion

There were very many people involved in training the UK team this year, and I thank them all. The performance of the students on the relatively easy questions is the reward for all that effort. Christopher Bradley and Adrian Sanders deserve special

acknowledgement for their protracted involvement. Christopher provided us with a thick wad of geometry questions before he went into purdah as a member of the problems group. During training the squad always relished the chance to work on these.

Gavin and Nathan missed out in Glasgow, but have another chance in Tokyo. Each of them is a very talented mathematician, and stands a good chance of making the team in 2003. We knew from taking so many practice examinations that occasionally even very good students can have a bad exam at IMO level. All the students worked extremely hard, and the atmosphere in the team was excellent. I am sure that I speak for Richard and all the other trainers when I say that it was a pleasure to work with all the students who formed part of the UK squad throughout the year. This includes those students who dipped in and out of the squad, all of whom made important contributions both mathematically and in terms of morale. The achievements of the UK team in Glasgow must be shared among these wonderful young people. It would not be appropriate to list all their names, but I will single out Erica Thompson from Scotland. She was trimmed from our squad after the FST at Cambridge. She filled the IMO sized gap in her life by getting selected to represent the UK at both Biology and Physics. She secured a gold medal at the IBO (coming 5th in the world) in Latvia and a bronze medal at the IPhO in Bali. In Glasgow the IMO team were following her progress by text messages from Indonesia ("now we are going to see a volcano").

Hannah Burton was a member of the UK IMO team in Washington. Every team at an IMO gets issued with a guide. Hannah took on that role in Glasgow, and looked after us very well. The social programme included excursions for the students, a trip on the Paddle Steamer Waverley on the Clyde estuary. On the last evening there was a party in a Glasgow night club (the Arches) which boasted a bucking bronco machine, a trampoline based game which involved hurling yourself upside down at a velcro wall wearing a sticky suit, and music played at volumes beyond the threshold of pain.

The IMO in Glasgow was a splendid event, and the new team uniform simplified dress issues at the ceremonies. The team met the Princess Royal in person, and each received a letter of support from the Prime Minister. Naturally the organization of such a large competition as the IMO places immense stress on the key organizers, but from the outside this was invisible. Angela Gould, Adam McBride, Peter Neumann and Robert Smart and their key advisers and assistants are to be congratulated on running such a marvellous event.