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PROBLEMS PROPOSED BY YUGOSLAVIA.

YU1 By $h(n)$, where n is an integer ≥ 2, let us denote the greatest prime divisor of the number n. Are there infinitely many numbers n for which $h(n) < h(n+1) < h(n+2)$ holds?

YU2 By $w(n)$, where n is an integer ≥ 2, let us denote the number of different prime number divisors of the number n. Prove that there exist infinitely many numbers n for which $w(n) < w(n+1) < w(n+2)$ holds.

YU3 Let S be a unit circle and K be a subset of S consisting of several closed arcs. Let K satisfy the following properties :

a) K contains three points A,B,C, which are the vertices of an acute-angled triangle;

b) for every point A which belongs to K its diametrically opposite point A' and all points B on an arc of length $\frac{1}{9}$ with centre A' do not belong to K.

Prove that there are three points E,F,G on S which are vertices of an equilateral triangle and which do not belong to K.

YU4 Prove that the functional equations $f(x+y) = f(x) + f(y)$ and $f(x+y+xy) = f(x) + f(y) + f(xy)$, $(x,y \in R)$ are equivalent.

YU5 Let $\mathcal{P}$ be the set of rectangular parallelepipeds which have at least one edge of integer length. If a rectangular parallelepiped P_0 can be decomposed into parallelepipeds $P_1, P_2, \ldots, P_n \in \mathcal{P}$, prove that $P_0 \in \mathcal{P}$