

THE XXI INTERNATIONAL MATHEMATICAL OLYMPIAD
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PROBLEMS PROPOSED BY FINLAND.

SF1 Show that for no integers $a \geq 1$, $n \geq 1$ is the sum
$$1 + \frac{1}{1+a} + \frac{1}{1+2a} + \dots + \frac{1}{1+na}$$
an integer.

SF2 For $k = 1, 2, \dots$ consider the k -tuples (a_1, a_2, \dots, a_k) of positive integers such that

$$a_1 + 2a_2 + \dots + ka_k = 1979.$$

Show that there are as many such k -tuples with k odd as there are with k even.

SF3 Show that for any vectors \bar{a}, \bar{b}

$$|\bar{a} \times \bar{b}|^3 \leq \frac{3\sqrt{3}}{8} |\bar{a}|^2 |\bar{b}|^2 |\bar{a} - \bar{b}|^2.$$