THE XXI INTERNATIONAL MATHEMATICAL OLYMPIAD
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PROBLEMS PROPOSED BY SWEDEN.

S1 Determine the maximum value of

$$x^2y^2z^2w$$

when x, y, z, w are ≥ 0 and $2x + xy + z + yzw = 1$.

S2 Given the integer $n > 1$ and the real number $a > 0$, determine the maximum of

$$\sum_{1}^{n-1} x_i x_{i+1}$$

taken over all nonnegative numbers x_i with sum a.

S3 Let $a_1 \leq a_2 \leq \ldots \leq a_n$ and $b_1 \leq b_2 \leq \ldots \leq b_n$ be two sequences such that

$$\sum_{k=1}^{m} a_k \geq \sum_{k=1}^{m} b_k$$

for all $m \leq n$ with equality for $m = n$. Let f be a convex function defined on the real numbers. Prove that

$$\sum_{k=1}^{n} f(a_k) \leq \sum_{k=1}^{n} f(b_k) .$$

S4 T is a given triangle with vertices P_1, P_2, P_3. Consider an arbitrary subdivision of T into finitely many subtriangles such that no vertex of a subtriangle lies strictly between two vertices of another subtriangle. To each vertex V of the subtriangles there is assigned a number n(V) according to the following rules:

(i) If $V = P_i$, then $n(V) = i$

(ii) If V lies on the side P_iP_j of T, then $n(V) = i$ or j.

(iii) If V lies inside the triangle T, then n(V) is any of the numbers 1, 2, 3.

Prove that there exists at least one subtriangle whose vertices are numbered 1, 2 and 3.

Fig.