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PROBLEMS PROPOSED BY ROMANIA

R1 Consider the sequences $(a_n),(b_n)$ defined by $a_1=3, b_1=100$, $a_{n+1}=3^{a_n}$, $b_{n+1}=100^{b_n}$. Find the smallest integer m for which $b_m = a_{100}$.

R2 Let a,b be mutually prime integers. Show that the equation $ax^2+by^2=z^3$ has an infinite set of solutions (x,y,z) with x,y,z integers and x,y mutually prime (in each solution).

R3 Show that, for every natural n, $n\sqrt{2} - [n\sqrt{2}] > \frac{1}{2n\sqrt{2}}$ and that for every $\varepsilon > 0$ there exists a natural n with $n\sqrt{2} - [n\sqrt{2}] < \frac{1}{2n\sqrt{2}} + \varepsilon$.

R4 Let M be a set and A,B,C be given subsets of M. Find a necessary and sufficient condition for the existance of a set $X \subset M$ for which $(X \cup A) \setminus (X \cap B) = C$. Describe all these sets X.

R5 Prove that there exists a natural number k_o such that for every natural $k > k_o$ we may find a finite number of lines in the plane, not all paralel to one of them, which divide the plane exactly in k regions. Find k_o.