THE XXI INTERNATIONAL MATHEMATICAL OLYMPIAD
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PROBLEMS PROPOSED BY POLAND.

PL1 Let be given m positive integers $a_1, \ldots, a_m$. Prove that there exist less than 2^m positive integers $b_1, \ldots, b_n$ such that all sums of distinct b_k's are distinct and all a_i $(i \leqslant m)$ occur among them.

PL2 Let ABC be an arbitrary triangle and let $S_1, S_2, \ldots, S_7$ be circles satisfying the following conditions :

S_1 is tangent to CA and AB,

S_2 is tangent to S_1, AB and BC,

S_3 is tangent to S_2, BC and CA,

....

S_7 is tangent to S_6, CA and AB.

Prove that the circles S_1 and S_7 coincide.

PL3 Let be given a real number $\lambda > 1$ and a sequence (n_k) of positive integers such that

$$\frac{n_{k+1}}{n_k} > \lambda \quad \text{for } k = 1,2,\ldots \quad .$$

Prove that there exists a positive integer c such that every positive integer n cannot be presented in more than c ways in the form $n = n_k + n_j$ or $n = n_r - n_s$.

PL4 An infinite increasing sequence of positive integers n_j /j=1,2,... .../ has the property that for a certain C and every $N > 0$

$$/*/ \quad \frac{1}{N} \sum_{n_j \leq N} n_j \leq C \; .$$

Prove that there exist finitely many sequences $m_j^{/i/}$ /i = 1,2,...,k/ such that

$$\{n_1, n_2, \ldots\} = \bigcup_{i=1}^{k} \{m_1^{/i/}, m_2^{/i/}, \ldots\}$$

and $m_{j+1}^{/i/} > 2m_j^{/i/}$ $/1 \leq i \leq k,\ j=1,2,\ldots/$.