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PROBLEMS PROPOSED BY THE NETHERLANDS.

NL1 Inside an equilateral triangle ABC one constructs points P, Q and R such that

$$\angle QAB = \angle PBA = 15^0 ,$$
$$\angle RBC = \angle QCB = 20^0 ,$$
$$\angle PCA = \angle RAC = 25^0 .$$

Determine the angles of triangle PQR.

NL2 In the plane a circle C of unit radius is given. For any line ℓ a number $s(\ell)$ is defined in the following way : if ℓ and C intersect in two points, $s(\ell)$ is their distance, otherwise $s(\ell) = 0$.
Let P be a point at distance r from the centre of C. One defines $M(r)$ to be the maximum value of the sum $s(m) + s(n)$, where m and n are variable mutually orthogonal lines through P.
Determine the values of r for which $M(r) > 2$.

NL3 Let there be given two sequences of integers

$$f_i(1),\ f_i(2),\ f_i(3),\ \dots \qquad (i = 1, 2)$$

satisfying

(i) $f_i(n,m) = f_i(n).\ f_i(m)$ if $g.c.d\ (n,m) = 1$,

(ii) for every prime P and all $k = 2,3,4,\dots$

$$f_i(P^k) = f_i(P)\ f_i(P^{k-1}) - P^2\ f(P^{k-2}).$$

Moreover, for every prime P

(iii) $f_1(P) = 2P$,

(iv) $|f_2(P)| < 2P$.

Prove that $|f_2(n)| < f_1(n)$ for all n.