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PROBLEMS PROPOSED BY ISRAEL.

IS1 Let a,b,c denote the lengths of the sides BC,CA,AB, respectively of a triangle ABC. If P is any point on the circumference of the circle inscribed in the triangle, show that $a\cdot PA^2 + b\cdot PB^2 + c\cdot PC^2 = \text{constant}$.

IS2 The non-negative real numbers x_1,x_2,x_3,x_4,x_5,a satisfy the following relations:

$$\sum_{i=1}^{5} ix_i = a$$

$$\sum_{i=1}^{5} i^3x_i = a^2$$

$$\sum_{i=1}^{5} i^5x_i = a^3 .$$

What are the possible values of a?

IS3 For any positive integer n we denote by $F(n)$ the number of ways in which n can be expressed as the sum of three different positive integers, without regard to order. Thus, since $10 = 7+2+1 = 6+3+1 = 5+4+1 = 5+3+2$, we have $F(10) = 4$.

Show that $F(n)$ is even if $n \equiv 2$ or $4 \pmod 6$, but odd if n is divisible by 6.

IS4 Let K denote the set $\{a,b,c,d,e\}$. F is a collection of 16 different subsets of K and it is known that any three members of F have at least one element in common. Show that all 16 members of F have exactly one member in common.