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PROBLEMS PROPOSED BY CZECHOSLOVAKIA.

CS1 Let S be a set of $n^2 + 1$ closed intervals (n positive integer). Prove that at least one of the following assertions holds:

(1) There exists a subset S' of $n + 1$ intervals from S such that the intersection of the intervals in S' is non-void.

(2) There exists a subset S'' of $n + 1$ intervals from S such that any two of the intervals in S'' are disjoint.

CS2 Let $n \geqq 2$ be an integer. Find the maximal cardinality of a set M of pairs (j, k) of integers, $1 \leqq j < k \leqq n$, with the following property:

$$\text{if } (j, k) \in M, \text{ then } (k, m) \notin M \text{ for any } m.$$

CS3 Let Q be a square with side of length 6. Find the smallest integer n such that in Q there exists a set S of n points with the property that any square with side 1 completely contained in Q contains in its interior at least one point from S.