THE XXI INTERNATIONAL MATHEMATICAL OLYMPIAD
LONDON 1979

PROBLEMS PROPOSED BY BRAZIL.

BR1 $M=(a_{ij})$, $i,j=1,2,3,4$, is a square matrix of fourth order.
Given that:

- For each $i=1,2,3$ and 4 and for each $k=5,6,7$

$$a_{ik}=a_{ik-4}$$
$$P_i=a_{1i}+a_{2i+1}+a_{3i+2}+a_{4i+3}$$
$$S_i=a_{4i}+a_{3i+1}+a_{2i+2}+a_{1i+3}$$
$$L_i=a_{i1}+a_{i2}+a_{i3}+a_{i4}$$
$$C_i=a_{1i}+a_{2i}+a_{3i}+a_{4i}$$

- For each $i,j=1,2,3$ and 4,

$$P_i=P_j$$
$$S_i=S_j$$
$$L_i=L_j$$
$$C_i=C_j$$

- and $a_{11}=0$

$a_{12}=7$

$a_{21}=11$

$a_{23}=2$

and $a_{33}=15$

Find the matrix M.

BR2 The sequence (a_n) of real numbers is defined as follows:

$$a_1=1 \quad , \quad a_2=2 \quad \text{and} \quad a_n=3a_{n-1}-a_{n-2} \quad , \quad n\geq 3$$

Prove that for $n\geq 3$,

$$a_n=\left[\frac{a_{n-1}^2}{a_{n-2}}\right]+1$$

where $[x]$ denotes the integer p such that $p\leq x\leq p+1$

THE XXI INTERNATIONAL MATHEMATICAL OLYMPIAD
LONDON 1979

PROBLEMS PROPOSED BY BRAZIL.

BR3

The real numbers $\alpha_1, \alpha_2, \alpha_3, \ldots\ldots, \alpha_n$ are positive numbers.
Let us denote by

$$h = \frac{n}{\frac{1}{\alpha_1} + \frac{1}{\alpha_2} + \ldots + \frac{1}{\alpha_n}} \qquad \text{the harmonic mean}$$

$$g = \sqrt[n]{\alpha_1\alpha_2 \ldots\ldots\alpha_n} \qquad \text{the geometric mean}$$

$$a = \frac{\alpha_1 + \alpha_2 + \ldots + \alpha_n}{n} \qquad \text{the arithmetic mean}$$

Prove that $h \leq g \leq a$, and that each of the equalities implies the other one.

SMP Westfield College, Kidderpore Avenue, Hampstead, London NW3 7ST Telephone 01-435 7141