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PROBLEMS PROPOSED BY BULGARIA.

BG1 Find all polynomials $f(x)$ with real coefficients, for whic

$$f(x)f(2x^2) = f(2x^3+x).$$

Variant 1

BG2 **Problem.** Prove that a pyramid $A_1A_2\dots A_{2k+1}S$ with equal lateral edges and equal space angles between adjacent lateral walls is regular.

Variant 2

Prove that a pyramid $A_1\dots A_{2k+1}S$ with equal space angles between adjacent lateral walls is regular, if there exists a sphere tangent to all its edges.

BG3 A pentagonal prism $A_1A_2\dots A_5B_1B_2\dots B_5$ is given. The edges, the diagonals of the lateral walls and the internal digonals of the prism are all coloured in either red or green, in such a way that no triangle, whose vertices are vertices of the prism, has its three edges of the same colour. Prove that all edges of the bases are of the same colour.

BG4 The plane is devided into equal squares by parallel lin i.e. a square net is given. Let M be an arbitrary set of n squares of this net. Prove, that it is possible to choose not less tha $n/4$ squares of M in such a way, that no two of them have a commo point.