THE XXI INTERNATIONAL MATHEMATICAL OLYMPIAD
LONDON 1979

TUESDAY JULY 3rd 1979

Time : 4 hours.

(4) Given a plane π , a point P in this plane and a point Q not in π, find all points R in π such that the ratio

$(QP + PR)/QR$ is a maximum.

(5) Find all real numbers a for which there exist non-negative real numbers x_1, x_2, x_3, x_4, x_5 satisfying the relations

$$\sum_{k=1}^{5} kx_k = a, \qquad \sum_{k=1}^{5} k^3 x_k = a^2, \qquad \sum_{k=1}^{5} k^5 x_k = a^3 .$$

(6) Let A and E be opposite vertices of a regular octagon. A frog starts jumping at vertex A. From any vertex of the octagon except E, it may jump to either of the two adjacent vertices. When it reaches vertex E, the frog stops and stays there.

Let a_n be the number of distinct paths of exactly n jumps ending at E. Prove that $a_{2n-1} = 0, \quad a_{2n} = \frac{1}{\sqrt{2}}(x^{n-1} - y^{n-1}), n = 1,2,3,\ldots,$

where $x = 2 + \sqrt{2}$ and $y = 2 - \sqrt{2}$.

* Note: A path of n jumps is a sequence of vertices $(P_0, \ldots, P_n)$ such that

(i) $P_0 = A$, $P_n = E$;

(ii) for every i, $0 \leq i \leq n - 1$, P_i is distinct from E;

(iii) for every i, $0 \leq i \leq n - 1$, P_i and P_{i+1} are adjacent.

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