IMO 2023 - UK team report

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The UK Mathematics Trust[2] organises competitions, mentoring and other enrichment activities for talented and enthusiastic school-aged mathematicians. One strand is a training programme for the country's top young problem-solvers to introduce them to challenging material, enjoyable in their own right, but also the focus of international competitions. Our goal is to use these competitions as a framing device to promote ambitions and excitement for mathematics and a positive scholarly culture amongst all children who attend our events, not just those who end up representing the UK on an international team.

The International Mathematical Olympiad is the original and most prestigious such event, now in its 64th edition. About a hundred countries are invited to send teams of up to six contestants to take part. This year's competition was held in Chiba, near Tokyo in Japan, with exams on July 8th and 9th.

Background and UK results

The UK is fortunate to be able to offer some training events leading to the IMO, and other international competitions. After the pandemic regulations had eased, and various internal difficulties were resolved, we were able to return to a full programme of in-person residential camps, including at Oxford in August 2022, in Budapest over New Year, and at Cambridge in April. The team of six students was selected via four 4.5 hour exams held in April and May, and trained on international-level problem-solving both before and after selection.

Here are the individual results of the UK team at IMO 2023:

	P1	P2	P3	P4	P5	P6	Σ	
Hanks Chong	7	3	0	7	3	0	20	Bronze Medal
Alex Chui	7	7	7	7	7	0	35	Gold Medal
Thomas Kavanagh	7	7	1	7	7	0	29	Silver Medal
Isaac King	7	7	4	7	7	0	32	Gold Medal
Sida Li	7	6	3	4	3	0	23	Bronze Medal
William Thomson	7	0	7	7	7	0	28	Silver Medal

In the (unofficial) team rankings, the UK came 13th, for the second year in a row. This represents an excellent team performance on these papers, especially given the prominence of geometry as Problem 2 and Problem 6, which is traditionally the UK's weakest subject.

At an individual level, this was Alex's second gold medal (along with two silvers, having represented Hong Kong in 2020 and 2021), which leaves him well on track to achieve one of the highest IMO records of all time before leaving school. In combination with Isaac, whose gold medal follows silvers in 2021 and 2022, this is the fourth time in eight years that the UK has had multiple gold medallists. It is also the seventh time in eight years that the UK placed in the top fifteen countries, and both of these achievements are to be celebrated; they would not have been possible without all the contributions to our training programme. Unlike many

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other countries around the IMO top ten, the UK's school curriculum offers only very limited preparation towards this kind of mathematical problem-solving, and both the volunteers and the students themselves have worked hard to bridge that gap.

Full results can be found at `https://www.imo-official.org/year_info.aspx?year=2023`.

A note on UKMT

Some readers will be aware that the UK Maths Trust faced a number of serious challenges in the period leading up to Autumn 2022.

There would be the option to draw a veil over these difficulties. But given how obvious some consequences were, it makes sense to comment briefly.

Over the academic year 2021/22, there were differences in opinion between managers and lead volunteers about how events should be organised and budgeted, and about how future strategy should be shaped. This discord eventually became irreconcilable as a number of events were cancelled, and others (including parts of the IMO programme) faced similar threats.

Happily, relations are now more collegial, and focus can shift back to running and improving our current suite of events, rather than existential concerns. We are particularly relieved that the Olympiad Mentoring Schemes will be returning after a one-year absence, and are glad that other UKMT activities affected by the pandemic will also now be restarting.

We share the disappointment of students who missed these activities in 2022/23, especially coming straight after the disruption of the pandemic period.

Commentary on the problems of IMO 2023

Overall, I was very satisfied with this pair of IMO papers, and in my subjective opinion it was potentially the best set of questions for (at least) ten years. It worked particularly well to have a balanced range of difficulty represented: P3 was amongst the easier 'hard' problems in recent history; while P4 and P5 were harder than usual, but not dispiritingly so. This avoided clumping of the marks, and gave all students, regardless of their experience levels, a chance to distinguish themselves. A handful of countries earned their first ever medal, or first ever honourable mention (for a perfect solution to a problem), and this was an ideal paper to allow scope for these achievements without compromising the standards of the contest.

These discussions of aspects of the problems are targetting a wide range of readers, and I hope there will be something of interest for everyone. Full solutions to the problems of IMO 2023 are easily found on the internet. Here, we discuss some steps of the solutions, and sometimes some background or tangents. For up-and-coming students interested in working towards the IMO, I hope that it's clear that trying the problems yourself is also useful.

Problem 1

Determine all composite integers $n > 1$ that satisfy the following property: if $d_1, d_2, \ldots, d_k$ are all the positive divisors of n with $1 = d_1 < d_2 < \cdots < d_k = n$, then d_i divides $d_{i+1} + d_{i+2}$ for every $1 \leqslant i \leqslant k-2$.

This was among the more approachable IMO questions in recent years, and would not be out of place on the first round of the British Mathematical Olympiad. But it still has plenty of value.

A general comment is that for this question, it might be useful to try 'small and specific' examples (like $n = 6, 8, 9, 10$ etc), but it's almost certainly more useful to try 'general but simple' examples. We are given that n cannot be prime[3], so the simplest candidates for the form of a non-prime n are:

- n is a *power of a prime*, so $n = p^k$ for $k \geqslant 2$;
- n is a *product of exactly two primes*, so $n = pq$ for p, q distinct primes.

In the power of a prime case, n satisfies the problem condition, since $p^\ell \mid p^{\ell+1} + p^{\ell+2}$. For the $n = pq$ case, it is important to clarify the order of the divisors, so we can assume 'without loss of generality' that $p < q$. Then the divisors are $1 < p < q < n$, and we note that $p \mid n$ but $p \nmid q$, so $p \nmid q + n$. So this form of n does *not* satisfy the problem condition.

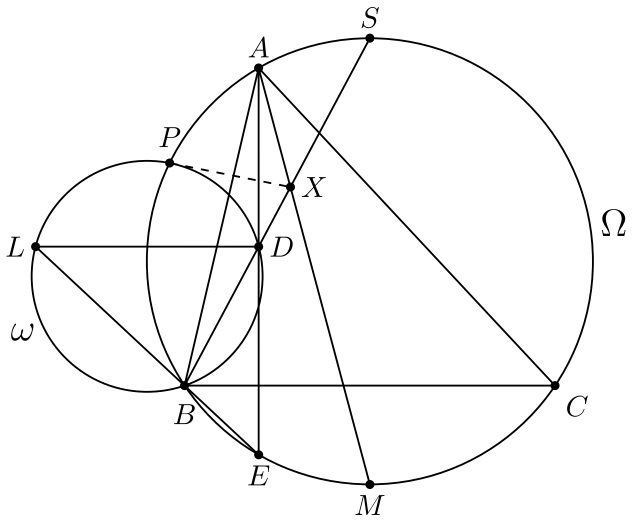
Of course there are many other forms for a composite integer n, but the second case has suggested a broad idea for a proof approach: find a divisor d_i such that d_i divides one of d_{i+1}, d_{i+2} but not the other one.

In fact, this approach can be made to work for all composite numbers that are not powers of primes (ie have at least two different prime divisors). It is tempting to introduce very general notation such as $n = p_1^{\alpha_1} p_2^{\alpha_2} \cdots p_m^{\alpha_m}$ but one should be cautious. Maybe this extra notation will obscure what is going on? It may be valuable to focus only on the smallest prime divisor. In any case, there are a handful of ways to extend this idea to all n that are not powers of primes, and we will avoid offering any further spoilers.

[3] If n is prime, the statement simply doesn't make sense - one needs at least three divisors before the given condition is meaningful.

Problem 2

Let ABC be an acute-angled triangle with $AB < AC$. Let Ω be the circumcircle of ABC. Let S be the midpoint of the arc CB of Ω containing A. The perpendicular from A to BC meets BS at D and meets Ω again at $E \neq A$. The line through D parallel to BC meets line BE at L. Denote the circumcircle of triangle BDL by ω. Let ω meet Ω again at $P \neq B$. Prove that the line tangent to ω at P meets line BS on the internal angle bisector of $\angle BAC$.



By the time of IMO Day One, the Problem Selection Committee had over ten qualitatively different solutions to this geometry problem, and more appeared over the following days. The UK students produced 4.5 solutions during the contest, which were indeed structurally different to each other even if they had some similar initial or final steps. It is a rich configuration and even without a perfect diagram (such as the software-drawn figure above) one might notice or speculate on some potential additional facts:

- L, P, S are collinear;
- $\angle APD = 90°$.

In general, for harder geometry, it's good to look for other features of the diagram before directly attacking the conclusion. As well as earning partial marks, these facts above start to give some ideas for how to study X (the intersection point). You will also notice that the diagram above includes M, the other arc-midpoint. One should be cautious about introducing extra points into the figure without a good reason, but in this case there are plenty of good reasons:

- M lies on the internal angle bisector of $\angle BAC$;
- MS is a further line perpendicular to BC (indeed, it is the perpendicular bisector), and thus also perpendicular to LD.

We'll finish by giving a hint towards what is perhaps the most elegant solution that we're aware of (which may also be the motivation for the configuration in the first place). Let the tangent to ω at P meet the circumcircle Ω again at point $Q \neq P$. Now focus on triangles $\triangle APD$ and $\triangle SMQ$, noting that $AD \parallel SM$. What else would you need to prove about these two triangles to finish the problem?

Problem 3

For each integer $k \geqslant 2$, determine all infinite sequences of positive integers $a_1, a_2, \ldots$ for which there exists a polynomial P of the form $P(x) = x^k + c_{k-1}x^{k-1} + \cdots + c_1 x + c_0$, where $c_0, c_1, \ldots, c_{k-1}$ are non-negative integers, such that

$$P(a_n) = a_{n+1}a_{n+2}\cdots a_{n+k} \tag{1}$$

for every integer $n \geqslant 1$.

It's not always clear what we should teach in an introductory session on using polynomials in olympiad problems. For new students, one of the hardest aspects is to develop an intuition for when it's best to think of a polynomial $P(x)$:

i) in its usual formulation from school via coefficients eg $P(x) = x^k + \cdots + c_1 x + c_0$;

ii) in terms of roots (possibly including complex roots) eg $P(x) = (x \pm \beta_1)(x \pm \beta_2)\cdots(x \pm \beta_k)$;

iii) as a general function $P\colon \mathbb{R} \to \mathbb{R}$ with various softer properties.

In school, the focus is often on passing from i) to ii), for example in solving a quadratic equation via various methods. In harder olympiad problems, one might have to use all three interpretations, but it might not be obvious which one to start with!

The intuition here comes from considering each term in the product $a_{n+1}a_{n+2}\cdots a_{n+k}$ as

$$a_{n+1} = a_n + (a_{n+1} - a_n), \quad a_{n+2} = a_n + (a_{n+2} - a_n), \quad \text{etc}$$

Sequences don't have to be particularly well-behaved. (*) They don't have to be increasing or decreasing, and they can have big jumps. However, many examples of sequences are somewhat well-behaved, and so it's helpful to consider that when n is large, we might have a_n large, and in particular, a_n much larger than the increments $a_{n+i} - a_n$. If so, we would write

$$\begin{aligned} P(a_n) &= (a_n + \beta_1)(a_n + \beta_2)\cdots(a_n + \beta_k) \\ &= (a_n + (a_{n+1} - a_n))(a_n + (a_{n+2} - a_n))\cdots(a_n + (a_{n+k} - a_n)), \end{aligned} \tag{2}$$

from which one might speculate that $a_{n+1} - a_n = \beta_1$ and $a_{n+2} - a_n = \beta_2$ and so on. Note that we aren't using the fact that $\beta_1, \beta_2, \cdots$ are roots of P, nor any consideration about whether they are real or complex or in fact integers, just the product decomposition. In any case, if this speculation were valid, then

$$a_{n+1} - a_n = \beta_1 = a_{n+2} - a_{n+1}, \tag{3}$$

which implies $\beta_2 = a_{n+2} - a_n = 2\beta_1$, and similarly $\beta_j = \beta_1$ for each $j = 2, \ldots, k$, and thus the sequence (a_n) is an *arithmetic progression* with non-negative common difference.

To move towards a formal argument, we have to use the given condition (1) to find some control over the sequence (a_n). The informal argument above relies on $a_n \gg a_{n+j} - a_n$ so it's reasonable to speculate that a_n is weakly increasing, or $a_n \to \infty$, and there are several ways to prove versions of these statements using (1). (In fact, one can even prove that a_n is either constant or strictly increasing.)

To make the argument at (2) work, we have to show that the increments $(a_{n+j} - a_n)$ are not too wild. Under the assumption that (a_n) is weakly increasing, we have

$$a_{n+1}^k = (a_n + (a_{n+1} - a_n))^k \leqslant P(a_n) = a_{n+1}a_{n+2}\cdots a_{n+k} \leqslant (a_{n+k})^k = (a_n + (a_{n+k} - a_n))^k.$$

Then, under the further assumption that $a_n \to \infty$, this shows that the increments $(a_{n+1} - a_n)$ are bounded, as otherwise for large enough a_n we would have large constants C satisfying:

$$(a_n + (a_{n+1} - a_n))^k \geqslant (a_n + C)^k > P(a_n).$$

We want to argue that for all n, it is true that $a_{n+j} - a_n = \beta_j$. It is useful to clarify that statements indexed by $n \geqslant 1$ can be true in the following senses:

a) Never true (ie true for no $n \geqslant 1$);

b) True for at least one value of n;

c) True for infinitely many n;

d) True for *all but finitely* many n;

e) True for all $n \geqslant 1$.

It's useful for improving students to be conscious when they see these kinds of assertions in different contexts, and keep an eye open for relations between them. For statements involving polynomials c) often implies d) or e) (eg a polynomial P with infinitely many roots is actually the constant zero polynomial). Statement b) remains true under finite unions ("one of the following is true...") and statement d) remains true under finite intersections ("all of the following are true"). In any case, using boundedness of the increments, the pigeonhole principle shows that we can choose (β_j) so that $a_{n+j} - a_n = \beta_j$ holds for infinitely many n. The question is whether this could also hold for a different collection (β'_j). But this would imply

$$P(x) = \prod_{j=1}^{k} (x + \beta_j) = \prod_{j=1}^{k} (x + \beta'_j),$$

which is only possible if (β'_j) is a permutation of (β_j). So in fact $a_{n+j} - a_n = \beta_j$ holds for *all but finitely* many n, ie for all $n \geqslant N_0$, some constant. This clarifies that (a_n) is *eventually* an arithmetic progression, using the argument at (3) and thus clarifies the form of the polynomial P. Lifting this to (a_n) always an arithmetic progression is a short final step.

Problem 4

Let $x_1, x_2, \ldots, x_{2023}$ be pairwise different positive real numbers such that

$$a_n = \sqrt{(x_1 + x_2 + \cdots + x_n)\left(\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n}\right)} \tag{4}$$

is an integer for every $n = 1, 2, \ldots, 2023$. Prove that $a_{2023} \geqslant 3034$.

I thought this was an outstanding olympiad problem for this difficulty level. The conclusion appears only loosely related to the given statement, so an initial task is to find any control at all on the growth of the sequence (a_n).

It is essential to use that the a_ns are positive integers. When replacing $n \mapsto n + 1$ in (4), the expression for a_{n+1} literally has more (positive) terms in it than the expression for a_n. So we have $a_{n+1} > a_n$ and then in fact we must have

$$a_{n+1} \geqslant a_n + 1. \tag{5}$$

By itself, this gives a bound of $a_{2023} \geqslant 2023$, which is about $2/3$ of the way to what we need. But to end up with $a_{2023} = 2023$, we would need equality to hold in (5) for every n. Since the a_ns are integers, there is a clear difference between equality holding, and equality not holding (unlike, say, when considering the equality cases for AM-GM or for Cauchy–Schwarz). That is

$$\text{either } a_{n+1} = a_n + 1, \quad \text{or } a_{n+1} \geqslant a_n + 2.$$

If equality holds for roughly half the values of n, then we would end up with $a_{2023} \gtrapprox 3034$. So we conjecture that the proof approach will involve studying when equality can hold in (5).

In fact, the key step is to show that we cannot have both $a_{n+1} = a_n + 1$ and $a_{n+2} = a_{n+1} + 1$. Thus indeed equality holds for at most 1011 of the values $n = 1, 2, \ldots, 2022$, leading to $a_{2023} \geqslant a_1 + 1011 \times 1 + 1011 \times 2 = 3034$.

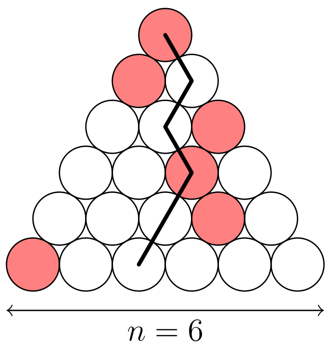
We won't explore the details of proving this key step, as most of the problem-solving element takes place before arriving at some version of this conjecture. However, again, a good piece of advice is to resist the temptation to use theory (especially AM-QM, which applies immediately to the quantity in (4)) until you are sure it is relevant. In this case, writing out

$$\begin{aligned} a_n^2 &= (x_1 + x_2 + \cdots + x_n)\left(\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n}\right) \\ a_{n+1}^2 &= (x_1 + x_2 + \cdots + x_n + x_{n+1})\left(\frac{1}{x_1} + \frac{1}{x_2} + \cdots + \frac{1}{x_n} + \frac{1}{x_{n+1}}\right) \end{aligned}$$

is more directly useful, since calculating $a_{n+1}^2 - a_n^2$ leads to lots of cancellation. It is useful to then aim for an algebraic condition equivalent to $a_{n+1} = a_n + 1$, and use this to show that $a_{n+1} = a_n + 1$ and $a_{n+2} = a_{n+1} + 1$ cannot hold simultaneously. As ever, it is important to use all the conditions given, which can be a hint[4] for how to end up with a contradiction!

Problem 5

Let n be a positive integer. A Japanese triangle consists of $1 + 2 + \cdots + n$ circles arranged in an equilateral triangular shape such that for each $i = 1, 2, \ldots, n$, the i^{th} row contains exactly i circles, exactly one of which is coloured red. A ninja path in a Japanese triangle is a sequence of n circles obtained by starting in the top row, then repeatedly going from a circle to one of the two circles immediately below it and finishing in the bottom row. Here is an example of a Japanese triangle with $n = 6$, along with a ninja path in that triangle containing two red circles.

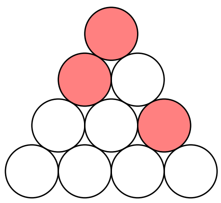


In terms of n, find the greatest k such that in each Japanese triangle there is a ninja path containing at least k red circles.

[4] Spoiler alert: one can derive that $x_{n+1} = x_{n+2}$ which contradicts the given conditions.

I thought this was a very suitable and pretty medium IMO question. For brevity let's call the number of red circles on a path the *weight* of that path. We are being asked to choose a red circle in each row so that the largest weight k of a ninja path is as small as possible. To do this requires both a construction, which provides an upper bound for k, and an argument to show that no smaller k is possible. In a 4.5 hour exam there is time to investigate both these avenues.

We will start with the more concrete avenue, the construction. A sensible and easy to analyse strategy (that is common in combinatorial problems) is to be *greedy*: at each stage make the choice that is currently best. Here that means, having already chosen the red coins in the top $i-1$ rows, we choose the red coin in the row i to minimise the largest weight of a ninja path on the top i rows. The top coin must be red and we may as well take the left coin in row 2 to be red. For row 3, making either the left or middle coin red gives a ninja path of weight 3 so we will be greedy and make the final coin red.



Now for row 4: every coin is in a ninja path of weight 2 and so regardless of which coin we make red there will be a ninja path of weight 3. We can make a choice and then continue to work greedily. As long as this is done sensibly (combinatorial constructions are normally quite regular) we will in fact get an optimal construction[5].

Reflecting on our method of construction, the quantity 'the largest weight of a ninja path from the top to a particular coin' was useful. Let's label each coin with this quantity (so in the figure above the top coin is labelled 1, the coins in row 2 are labelled 2 and 1, respectively). These labels are easy to analyse as we go from one row to the next: the label of a coin only depends on the labels of its two parent coins and whether it is red. The largest weight of a ninja path is the largest label in row n. There are various ways to control the maximum label in row i. Seeing how the labels vary on the construction may help suggest why this maximum is forced to increase as we descend the triangle.

[5]Spoiler alert: the largest weight k in the construction should satisfy $2^{k-1} \leqslant n < 2^k$

Problem 6

Let ABC be an equilateral triangle. Let A_1, B_1, C_1 be interior points of ABC such that $BA_1 = A_1C$, $CB_1 = B_1A$, $AC_1 = C_1B$, and

$$\angle BA_1C + \angle CB_1A + \angle AC_1B = 480^\circ.$$

Let BC_1 and CB_1 meet at A_2, let CA_1 and AC_1 meet at B_2, and let AB_1 and BA_1 meet at C_2. Prove that if triangle $A_1B_1C_1$ is scalene, then the three circumcircles of triangles AA_1A_2, BB_1B_2 and CC_1C_2 all pass through two common points.

(Note: a scalene triangle is one where no two sides have equal length.)

This is very hard and only ten students scored 6 or 7 on the problem. I don't have many comments to make beyond discussing how one might prove that three circles all pass through two common points. The most direct way would be to identify two distinct points and show that these lie on all three circles. In practice it can be hard to identify points on the circles. *Radical axes* provide a convenient alternative method that is relevant here.

Fix a point P and a circle ω. Let ℓ be any line through P that meets ω and call the points of intersection A and B. The quantity $PA \cdot PB$ is called the *power of P with respect to ω* and crucially, by the intersecting chords theorem, is independent of the line ℓ. This is a signed quantity which is negative if P is inside ω, zero if P is on ω, and positive if P is outside ω.

If there are two circles ω_1 and ω_2, then one can consider the locus of points that have the same power with respect to ω_1 as with respect to ω_2. This locus is called the *radical axis of ω_1 and ω_2*. Provided ω_1 and ω_2 do not share the same centre, their radical axis will be a line perpendicular to the line joining their centres[6]. If the two circles meet, then this line must pass through the two points of intersection (as these both have power zero with respect to the circles).

For three circles ω_1, ω_2, and ω_3 (as we have in this problem) we can consider the three radical axes that each pair create. Usually these would be three lines that concur at a point which has equal power with respect to all three circles. However, if the centres of the circles are collinear, then the radical axes will be parallel or coincide. The coinciding case is the relevant one here. If the radical axes coincide and the circles intersect properly, then the points of intersection must be common to all three circles and so the circles pass through two common points. The heart of this problem is showing that the radical axes coincide (it is not hard to see that the circles intersect properly). We can do this by finding two distinct points that have equal power with respect to all three circles.

There is one point that is easier (though still hard!) to find on the common radical axis, which earned 2 marks. Identifying a further point required quite a bit of imagination and skill.

[6] One can prove this, for example, by using Cartesian coordinates. First prove that the power of P with respect to ω is $OP^2 - r^2$ where O is the centre of ω and r is its radius (make a suitable choice for ℓ).

Brief IMO diary

pre-IMO training, Fuchu

Since 2007, the UK has held a joint training camp directly before the IMO with the Australian team. This habit was disrupted by the online IMOs of 2020 and 2021, and by the fraught global travel landscape of 2022. It was excellent to be able to rekindle this practice in 2023.

Each morning, the two teams sat a practice paper with the same format as the IMO (4.5 hours to attempt three problems) which we marked and discussed afterwards. Many of the problems were drawn from those shortlisted but not eventually used at last year's IMO 2022 in Norway. This is one of the best chances for the team to get relevant feedback and work on their write-up style. At the IMO, it is not enough to have all the right ideas; they have to be communicated clearly and rigorously!

In the middle of the week, to add some variety, the UK team set an exam for the Australians, and vice versa. Each team thus got some practice at the 'other side of the process', with first-hand experience of designing a markscheme, then grading solutions and semi-formally defending these grades. On the final day of the camp, the final practice exam was designated as the *Mathematics Ashes*, with an urn full of charred geometry solutions to be awarded to whichever team was victorious. This year, the timing was coincident with the other Ashes back in England[7]. There was significantly less controversy and media attention for our contest, but the same outcome - a draw, this time 89 points for both teams.

The afternoons offered some opportunities for light tourism. For many of the UK team, this was their first visit to Asia, and the sights and sounds (and temperature!) of the general urban landscape were just as interesting as specific attractions. Nonetheless, a visit to a *dojo* where they were taught how to use a traditional *katana* sword was a fascinating highlight. Our visit was well-timed with the city festival, where the nearby temple complex was illuminated with traditional red lanterns, and a fireworks display lasting almost an hour!

Leaders' phase - problem selection

As usual, the leaders gathered separately to select the problems for the paper from a wide-ranging shortlist. A number of other UK volunteers were present, including Geoff Smith as member of the IMO Board, Sam Bealing as a geometry expert on the Problem Selection Committee, and Vesna Kadelburg as a geometry coordinator.

This year's shortlist has a number of excellent problems on it. I enjoy spending some time in my room with an excellent view over the hive of activity that is the runway at Narita airport and working my way through the collection. We will surely use several of the remaining problems in our selection exams and training next year.

The process to choose the problems for the papers goes smoothly. There is very strong support for the problems which end up as P1, P5 and P6, which makes the remaining choices relatively straightforward, though many opinions are discussed. In fact, there is an award (the *golden microphone*) for the leader who makes the most speeches. I am determined not to repeat my 'success' in this competition from 2019 - fortunately many others have plenty to say, including our recent double-IMO-gold-medallist Yuka Machino, now a high-flying sophomore at MIT, and leader of the Ghanaian team. In the end, Yuka will finish as one of the runners-up behind Vincent from France.

[7] `https://www.espncricinfo.com/series/the-ashes-2023-1336037`

The English language committee meets to produce a final version of the paper. Many interesting points of mathematical and syntactical grammar are discussed. As chair, I run a tight ship, and decline the opportunity to rewrite P5 in terms of *samurai*. This year there are 112 participating countries, and the paper needs to be translated into versions for over 55 languages! This is understandably time-consuming, and allows the English-speakers a moment to slip away.

I make it into Tokyo, and find myself at the remarkable six-way Shibuya scramble crossing, and then the nearby Pokémon superstore! At both, the crowds are extraordinary, and being over 6 foot tall turns out to be very atypical and a huge advantage...

Chiba - coordination and excursions

After the second day's paper, the leaders are moved to Chiba, where the students sat the contest. They are staying in a very panoramic hotel with glorious views over Tokyo Bay. Most of the team are pleased with how they have done, and our initial impressions of their work are very encouraging. We are able to make a short sunset visit to the nearby beach, before dinner in a Thai restaurant, which is a welcome break from the tasty-but-repetitive buffet.

Freddie and I read the work in more detail, and expect to have no marks on P6, but no major issues on P1, P2, P4 and P5. Over the next two days, we meet with the coordinators to discuss the marks. The coordinators ensure that the marks awarded are consistent across countries and languages. We are particularly pleased with P5 - this complex problem has many steps where one could slip up in logic or in exposition, but all six students were clear about what they managed to do, and we get our requested marks without any discussion.

Problem 3 is more complex, as we have two solutions that are highly non-standard. Isaac in particular requires careful attention as he has demonstrated a very weak condition on the sequence (a_n). This causes problems in his argument later; however, parts of the argument are very clear and would work just fine if he were using a stronger condition on the sequence. The markscheme, perhaps unexpectedly, allows scope for this sort of mixed approach, and I was happy that a fair outcome was reached, though not without considerable work by everyone! We suspect that Isaac would only have needed another few minutes to reorder his pieces to turn this 4/7 into a 6 or 7 instead.

In any case, the scores are what they are. Sida was aiming high, and will be disappointed not to have made more of a splash on Day Two, but this can happen under the pressure of the IMO. Thomas and William are very pleased to have four perfect solutions each; and as a team we have clearly done particularly well on Day One. Isaac has to wait anxiously to find out whether 32 will be enough for a gold.

Fortunately, the students have had plenty of activity to fill their time while we have been working hard on their scripts. On Monday, they go to Disneyland. Apparently there is mixed interest in the more extreme rides, but everyone found something to enjoy. On the Tuesday, they visit Tokyo Zoo, and get a chance to see the sprawling metropolis from the top of the 600m SkyTree. Freddie and I finish our coordination in time to meet the team to visit the temple complex at Asakusa, and the pop culture suburb of Akihabara. It is 36° and spirits are wilting slightly in the humidity; fortunately, our excellent team guide, Chad, is on hand to lead the hunt for an ice cream.

Final moments

The final jury meeting later that evening includes discussion of future IMO hosts. It is great that the olympiad is now back in a position where many offers are made several years in advance. Next up will be IMO 2024 back in Bath, but after that the air miles will keep racking up, with visits to Australia and to Shanghai. Eventually we pass to the medal cutoffs where, exceptionally, it is voted to award slightly more than half the contestants a medal, rather than (as the rules state as default) noticeably less than half.

In the end, the gold cutoff is exactly 32, so the UK earns two golds, two silvers and two bronzes. Hanks is pleased with his bronze, as a stepping stone to two further chances; while Isaac's joy seems to transcend the usual bounds of human emotions. The team disappear off into the vast hotel complex to continue the celebrations with all the new friends they've made from around the world.

The following morning, there is time for a final visit for shopping for some distinctive Japanese souvenirs. The novelty flavoured Kit Kats are particularly popular! Then, to the closing ceremony, where medals are presented. Alex is in one of the final groups of gold medallists and gets a particularly loud round of applause. The UK team reappears on stage subsequently for the ceremonial transfer of the IMO flag from this year's hosts to next year's. Fortunately we have a generous baggage allowance on our flight tomorrow to get this back to Heathrow, ready for IMO 2024 in Bath!

Conclusion

Taking a UK team to the IMO requires a lot of work by many people. Thanks are particularly due to:

- Kit, Natalie, and all the staff at UKMT, for help in booking all the travel, and organising all the programme events during the year.
- Our Australian colleagues, Angelo, Ben, Louise, Hadyn and Michelle, with whom we ran a great training camp, and enjoyed sharing valuable ideas and experiences. We also thank Noriko Machino, who helped us so much in organising our training camp in Fuchu, and with everything from the challenge of finding genuinely vegetarian food to translating sword-fighting instructions! It was a fantastic week that left both the UK and Australian teams perfectly set for the competition.
- The organisers of IMO 2023, who put on a smooth-running and enjoyable competition in Chiba. We were particularly grateful for their help in meeting our specific access requirements so thoughtfully. We also compliment the problem selection process, which was outstanding this year, and gives Bath 2024 a high standard to aim for.
- The UK team's guide Chad Dudash was brilliant throughout, and helped tirelessly both with the logistics, and with showing the UK team an authentic side of Japanese culture during our excursions. We wish him the best for the rest of his time working in Japan.
- All the mathematicians, young and old, who gave sessions at our training camps through the year. After so much disruption, it was wonderful to have everyone together in person again, and all the students in the UK IMO programme benefitted from meeting such a rich collection of topics and mathematical perspectives.
- Ava Yeo, who looked after the team on the trip with kindness and humour, even when things didn't go perfectly, and helped them to make the most of the chances to explore Tokyo after the papers.
- Freddie Illingworth, who was an excellent Deputy Leader at the IMO again, as well as all the work training the team and writing preparation material through the year. He was outstanding at parsing even the most oddly-written work during coordination, and meant the UK was one of the only top teams able to manage coordination with just two academic staff. I'm also grateful for his comments on P5 and P6 from earlier in this report!
- Finally, of course, our UK team comprising Hanks, Alex, Thomas, Isaac, Sida, and William. They worked very hard to prepare for this competition, and we were delighted to see their success at IMO 2023. We look forward to seeing what Hanks, Alex and Isaac can do next time round, and we wish Thomas, Sida and William all the best as they move to university in the autumn to start the next step of their mathematical journey.