Balkan Mathematical Olympiad 2016 - UK Report

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The Balkan Mathematical Olympaid is a competition for secondary school students organised annually by eleven countries in Eastern Europe on a rotating basis. The 2016 edition was held in Tirana, Albania from 5$^{\text{th}}$ until 10$^{\text{th}}$ May. The UK was invited to participate as a guest nation. We have a self-imposed rule that students may attend this competition at most once, so that as many as possible might enjoy the experience of an international competition. This year's UK team was

Jamie Bell	King Edward VI Five Ways School	(17)
Rosie Cates	Hills Road Sixth Form College	(16)
Jacob Coxon	Magdalen College School	(17)
Michael Ng	Aylesbury Grammar School	(16)
Thomas Read	The Perse School	(17)
Renzhi Zhou	The Perse School	(18)

Gerry Leversha was deputy leader, and Jill Parker accompanied the students. This report mostly concerns matters mathematical, but a more light-hearted diary detailing what we got up to when we were not solving problems can be found in a pair of blog posts[1].

The results of the UK team were:

	P1	P2	P3	P4	Σ	
Jamie Bell	10	0	10	2	22	Bronze Medal
Rosie Cates	10	1	10	0	21	Bronze Medal
Jacob Coxon	10	1	9	0	20	Bronze Medal
Michael Ng	10	9	10	1	30	Silver Medal
Thomas Read	10	10	9	0	29	Bronze Medal
Renzhi Zhou	10	10	10	0	30	Silver Medal

The cutoffs for bronze, silver and gold medals were 17, 30 and 32 respectively. These were calculated with reference to the 62 contestants from official member countries, with roughly 2/3 of such contestants receiving a medal, and colours divided in the ratio 3 : 2 : 1. This is only the second time that all six contestants in the UK team at this competition

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[1] http://eventuallyalmosteverywhere.wordpress.com

have received medals, and reflects the strength throughout our training programme. At least one member of the UK team at this competition will also go to the International Mathematical Olympiad this July in Hong Kong, and this performance bodes very well for our upcoming selection tests and the IMO itself.

The leading team totals were (with guest nations in brackets): Serbia 181, (Kazakhstan 181), Romania 180, Turkey 172, Bulgaria 170, Greece 161, (UK 152), (Italy 150), (Saudi Arabia 145), Bosnia and Herzegovina 129. This continues the fluctuating fortunes of the leading participating nations: there have now been four winners of the past five Balkan olympiads, It is interesting to note that Kazakhstan win the consistency award, having earned 179/180 on the first three problems, thus giving them a sample variance of ~ 0.97.

Congratulations are due to the two contestants, from Serbia and Romania, who achieved a perfect score of 40/40, via a correct solution to the challenging final problem. The 12-year old participant from France also received a bronze medal so doubtless we will watch out for his progress over the coming years. It was also excellent to see eight countries represented among the gold medals, including two for Saudi Arabia, continuing the remarkable invigoration of competition mathematics there over the past few years.

The problems

The Balkan MO has a slightly different structure to the IMO. There is only one 4.5 hour paper, which contains four problems. The aim is that the first question will be accessible to all contestants and solved by many; middle questions will challenge those in contention for medals, and the final question will ideally be both difficult and beautiful.

These were the problems:

1. Find all injective functions $f : \mathbb{R} \to \mathbb{R}$ such that for every real number x and every positive integer n,

$$\left| \sum_{i=1}^{n} i\Big(f(x+i+1) - f(f(x+i))\Big) \right| < 2016.$$

(FYROM)

2. Let $ABCD$ be a cyclic quadrilateral with $AB < CD$. The diagonals intersect at the point F and lines AD and BC intersect at the point E. Let K and L be the projections of F onto sides AD and BC respectively, and let M, S and T be the midpoints of EF, CF and DF respectively. Prove that the second intersection point of the circumcircles of triangles MKT and MLS lies on the segment CD.

(Greece) Silouanos Brazitikos

3. Find all monic polynomials f with integer coefficients satisfying the following condition: there exists a positive integer N such that p divides $2\,(f(p))!+1$ for every prime $p>N$ for which $f(p)$ is a positive integer.

   Note: A monic polynomial has leading coefficient equal to 1*.*

   (Greece) Silouanos Brazitikos

4. The plane is divided into unit squares by two sets of parallel lines, forming an infinite grid. Each unit square is coloured with one of 1201 colours so that no rectangle with perimeter 100 contains two squares of the same colour. Show that no rectangle of size 1×1201 contains two squares of the same colour.

   Note: Any rectangle is assumed here to have sides contained in the lines of the grid.

   (Bulgaria) Nikolay Beluhov

Commentaries on the problems

The following commentaries on each problem are not supposed to be official solutions, though they do include solutions, or substantial steps of solutions. I've tried to emphasise what I feel are the key ideas, and how one might have arrived at them naturally, though both stages of this are highly subjective. Indeed, it was very interesting to discuss the problems, and later the solutions with the other leaders, as interpretations of the difficulty and style of many things vary from region to region, and between individuals.

Anyone hoping to try the problems themselves (especially any potential olympiad contestants) would be advised to skip this section, though the first half of each commentary might constitute a well-developed hint.

Problem 1

We should start by trying to get a working heuristic for what the inequality actually *means* about the function f, and forget about injectivity until later, because this property seems unrelated to the rest of the statement. I think it is fairly clear that the inequality is telling us that the quantities $f(x+1)-f(f(x))$ are small, in some bizarre sense. Certainly by taking $n=1$, we know they are less than 2016. But also by taking n large, we have a lot of these quantities, most pre-multiplied by large factors, so unless there's a good reason for all the large positive contributions to cancel the large negative contributions, we will struggle to get the stated inequality to hold unless $f(x+1)-f(f(x))$ is small, fairly uniformly.

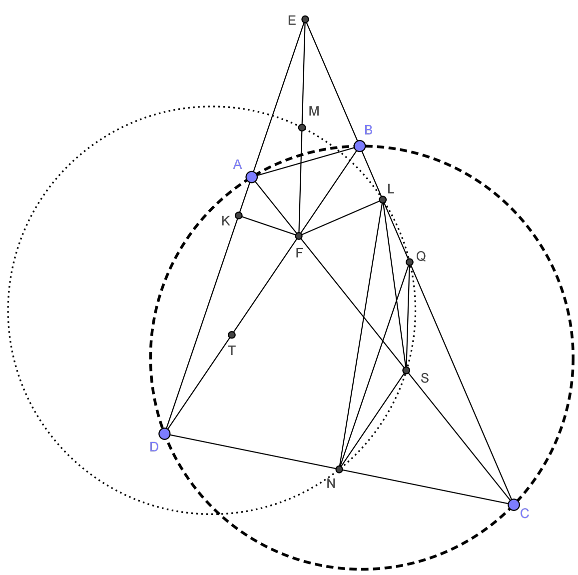
Fortunately there's a sum here, so we can isolate an individual term $f(x+1) - f(f(x))$ by looking at the sum for n and for $n-1$. Precisely

$$n(f(x+n+1)-f(f(x+n)) = \sum_{i=1}^{n} i(f(x+i+1)-f(f(x+i))-\sum_{i=1}^{n-1} i(f(x+i+1)-f(f(x+i)),$$

and thus by the triangle inequality, $n(f(x+n+1) - f(f(x+n))) < 2 \cdot 2016$. But we can take x to be whatever we want, so we conclude that $f(y+1) - f(f(y)) < \frac{2 \cdot 2016}{n}$, and since the LHS no longer depends on n, the only option is that $f(y+1) = f(f(y))$ for all y. Now it clearly *is* time for injectivity, which gives $f(y) = y+1$ immediately.

Problem 2

Showing that two circles and a line intersect at a single point might well be hard. The most natural thing to consider is showing that one of the circles meets the line at a point on that line which is independently defined. There are various contenders for what this point N might be, and drawing an accurate diagram will be essential for deciding what it is. So when we look at the following diagram, courtesy of Silouanos Brazitikos who composed the problem, and conclude that N should be the midpoint of CD, we shouldn't take this for granted. Making this inference during an actual competition is non-trivial and an important stage of any attempt.



Let's start with the circle through M, L and S. This circle goes through a pair of midpoints and the foot of a perpendicular, which reminds us of the nine-point circle[2]. Indeed, this is the nine-point circle of $\triangle EFC$. And, at the risk of being glib, the whole idea of the nine-point circle is that there are six other points on it. In this context, it's natural to add the midpoint of EC, since we already have lots of midpoints, and in this diagram we call it Q. (Indeed, this might have been our motivation for guessing that the relevant point N was the midpoint in the first place.)

So it remains to prove that N also lies on this circle. Since N doesn't really have anything to do with M, it's probably going to be easiest to prove L, Q, S, N are concyclic, being careful about orientation[3]. I don't really have very much to say about about how to do this, except that LS has a natural interpretation in $\triangle FLS$.

Alternatively, our contestant Michael considered expanding with scale factor 2 from C. This takes the nine-point circle to a new circle, which goes through more of the original points, but where the image of M is more awkward. This turned out not to be a problem at all, and I leave this hint for the reader to consider.

The keen observer might have focused on the parallelogram FSNT, and noticed that with respect to $\triangle MST$, this is the same configuration as appeared in Q2 of this year's EGMO in Romania (which was itself similar to an IMO shortlist problem from 2012). Spotting this might prompt an investigation of the angles $\angle TMF$ and $\angle NMS$, which turn out to be equal. I increasingly feel that spotting common configurations hiding inside more complicated figures is a key step in olympiad geometry, and am actively thinking about how to incorporate more of this sort of technique in our training.

Problem 3

This is a classic competition problem. An unusual statement is given, and you can have to get a handle on how to use each of the conditions you have. In particular, where will we use the fact that f is monic, and the fact that the statement holds for all but an initial segment of primes?

What about when $f(p)$ is not a positive integer? The only other option is for $f(p)$ to be a negative integer or zero, and since f has a positive leading coefficent (i.e. it's 1), this won't be an issue once we start looking at large values of p.

Once we have a range of p large enough that the statement is defined, we might think about when p divides a factorial, and indeed when it doesn't divide a factorial, since the latter is what we have here, albeit with a bit more information. We know that $p|n!$ exactly

[2] Elsewhere, this is commonly called the *Euler circle*.
[3] For example, one might use directed angles throughout.

when $n \geq p$, so we conclude that $f(p) < p$ whenever both the statement makes sense and we are above the threshold N.

But f is a polynomial with positive leading coefficient, so this can only happen if f is linear[4], that is $f(x) = x - a$. So we require

$$2(p-a)! \equiv -1 \mod p,$$

for all large enough p. The form of this relation encourages us to think about Wilson's theorem, which asserts that

$$(p-1)! \equiv -1 \mod p,$$

so we would require

$$(p-a+1)(p-a+2)\cdots(p-2)(p-1) \equiv 2 \mod p.$$

It's very hard to get a grip on the LHS in this form, but fortunately $p - b \equiv -b$ modulo p, so we get

$$(a-1)(a-2)\cdots 1 \equiv \pm 2 \mod p. \quad (1)$$

We could decide whether it is 2 or -2 in terms of the parity of a (which, recall, is fixed), but this is a distraction. Instead, we remark that even though we have already used the fact that this holds for infinitely many p, that doesn't mean we can't use it again! So if we take p very large, in particular, much larger than the LHS of (1), then we can replace modular equivalence with equality:

$$(a-1)! = 2.$$

So $a = 3$, which leads to $f(x) = x - 3$ is the only such polynomial.

Problem 4

When I read this problem, I thought two things. Firstly, why 1201? Secondly, if the required conclusion is true, then maybe we can say even more? Chasing this second thought, if the conclusion holds, then in any 1201×1201 subgrid, we must have each colour represented exactly once in every row and column, as in a Latin square. This is a strong conclusion, because it means that the colouring is periodic, since the 1202^{nd} column must be identical to the 1^{st} column, and so on.

So we know that the condition about rectangles with perimeter 100 is actually enough to ensure that the colouring is periodic. Maybe we will end up with further restrictions on

[4] There was variation in opinion about how clear this was. I think it's very clear. If f has higher degree, it eventually dominates any linear function, including the identity.

what can happen within the period, but this feels strong, so my plan was to make sure that all my initial steps were strong too.

My first thought was to colour the square at the origin blue, and look at where I was now banned from placing blue squares. You get a diamond with width and height 97, and the number of cells in such a diamond is 4705, which didn't feel related to 1201 very much. I also wasn't sure what to do next. Where would the nearest blue neighbours to my origin actually be? This didn't feel like a strong initial step, so I abandoned it. However, I did notice that if instead I took a diamond with width 49, it would contain 1201 cells. This has suddenly turned into a potentially strong observation, while also answering my first initial question about the problem, namely 'where does 1201 come from'?

Once you know what shape to look in, given the perimeter 100 condition of the question it's fairly clear that this diamond with width 49 must contain every colour exactly once (*). This definitely is a strong conclusion, because we aren't throwing away any numbers, and diamonds can tesselate the plane. So we can guess what the set of legal colourings is going to be: colour a diamond with each colour exactly once, then tesselate the plane with coloured copies of the original diamond (**).

The markscheme decided that what remained was the body of the question, but I feel we've done the hard part, by finding a strong reason why this apparently unrelated number 1201 appeared, even though it would be short to write up. The remainder of the question is a good technical exercise. To prove that versions of (**) are the only valid colourings, you need to fix a colour, say blue, and show that the condition (*) applied on various diamonds is enough to force the blue squares to form a lattice corresponding to (**). Finally, you need to show that this fairly exact knowledge of the structure of the colouring (**) implies the condition we were required to prove. It's very easy to argue this confusingly or erroneously, but it is really just an exercise in notation for the lattice of squares of any given colour.

Overall though, I thought this problem was fantastic, as the condition was very surprising, but responded well to playing around with diagrams. Then, once the key idea had been found, there was still some work to do, but this work could be done in small, careful stages.

The stages of the contest

Problem selection

The leaders meet in central Tirana on Thursday night, and receive a booklet of 19 problems, proposed by the member countries. It seems unclear this year whether the UK has proposed any problems, nor whether such proposals are statutorily acceptable. I spend some of the evening thinking about some of these. Fortunately, daylight hours in Albania feel odd to a visitor from the UK - it is in the same time-zone as Spain - and so I also spend a substantial part of the morning thinking about some more.

There's a noticeable difference in topic area preference. Symmetric and almost-symmetric inequalities feel overwhelmingly out of favour at the IMO, and have thus not been featuring heavily in the UK training recently. More problems here seems to turn on specific pieces of theory too. It's tempting to object to this on the grounds that it places less well-trained students at a disadvantage, which is undoubtedly true, but I feel the rewards can justify the cost. Most of the member countries represented at this competition have a mathematical education system which means that almost all the contestants will have met elementary number theory and lots of Euclidean geometry at school. We teach Wilson's theorem at our camps, mostly because it is natural, elegant, and a good way to reinforce previous theory, but it's nice to see it feature as a step in a contest in Q3 here.

Overall, there are several gems, and a few anti-gems on the shortlist. I make a list of my favourite problems, and they are all in two topics areas. One of the other areas is light on questions altogether, and the final area has plenty of problems, but most seem to me very hard or quite contrived. The good problems are excellent though, and I hope some of them make it onto the paper. I write down a draft of my ideal paper, based on a mixture of what I find interesting, and what I think the UK students will enjoy. Since I have set most of the training material recently, possibly it's unsurprising that these qualities feel strongly positively correlated. I then turn my attention to the other problems, so that when they come up, I can at least vaguely look like I know what I'm talking about.

Most of Friday is spent with the other leaders choosing the paper. The only dramatic moment comes when the Greek leader flourishes a webpage and an old IMO shortlist problem which does indeed contain a proposed geometry problem as a lemma, and so it is rejected. Partly as a result of this, a medium geometry problem is chosen quickly; and the hard combinatorics problem shortly after lunch, on the convincing grounds that it is the favourite hard problem of literally every single leader! Selecting the final two problems, from number theory and algebra produces several combinatorial challenges in its own right. A rather complicated, multi-round election takes place (in which the UK, as a guest nation, does not get a say), and the final two problems are chosen, and the paper is complete.

Interestingly, this paper is exactly the one that I'd hoped for late the previous night. The only difference is that I placed the geometry at Q3, and the number theory at Q2. The scores of the UK students support my view, but the scores of several other countries' students support their respective leaders' views too. I find it interesting that other countries feel an overt weakness at combinatorics is a common phenomenon. It seems to be the prevailing view in the UK that one can either be weak at geometry, or an all-rounder. In any case, it makes for interesting dinner conversation with the leaders of Bosnia, FYROM and Montenegro.

I am summoned to be an expert on the usage of English to prepare the final version of the paper. I feel that the problem authors have done an excellent job, and there is little work to do except suggest some extra sentence breaks and delete some appearances of the word 'the'. Ultimately, the jury received hardly any queries from contestants about the meanings of the questions, except for the definition of 'injective' and 'projection', so a good job was done by all, especially the other leaders, who stayed up late translating and approving all the versions in their respective languages.

Overall, I think that the medium questions, in either order, are slightly easier than is typical for this competition. Two questions turn on taking some quantity to be arbitrarily large, but that is a fairly subtle similarity, and also quite a common thing to try in all kinds of mathematics. The fourth question is hard in contest circumstances, but that will make it even more rewarding for any contestant who solves it.

The contest

The exam takes place at the Harry Fultz institute, a high school in Tirana. The main building is many stories high and, despite its educational purpose, the exterior is entirely covered by a billboard advertising a local beer. I hope this does not reflect the nature of the refreshments offered to the contestants during the $4\frac{1}{2}$ hours of the competition paper.

The contestants may ask questions of clarification during the first half hour. Twenty-five minutes pass, and we are untroubled, so we smugly conclude we must have achieved a wording with total clarity. In fact, the exam is starting slightly late, and a mild deluge begins, mostly concerning the definition of 'injective'. Both the era of UK students asking joke questions and UK students asking genuine questions have passed, so I am left in peace.

Somehow, Enkel Hysnelaj has single-handedly produced LaTeX markschemes for all four problems overnight, and these are discussed at some length, though it's to his credit that they didn't require even longer. Their is an excursion planned to Krujë, famous as the hometown of Skenderbeg, the Albanian national hero, and just before that is Fushë-Krujë, famous as the place where George W. Bush's watch was stolen during an official visit. Despite the general flexibility with timetables, this starts to loom as the scheme for Q4 is discussed, and we have a situation where making a substantial step will not be richly rewarded, but hopefully this will not affect many students substantially.

As the leaders are not around to greet the contestants after the exam, they tell me their progress via texts. It sounds like everyone has solved questions 1 and 3, with three claimed solutions to the geometry, and some non-trivial progress worth reading on the final hard problem. There's a big difference between 15 full solutions, and 15 well-written full solutions. The scripts arrive after an impressive turnaround time and it's clear we have the latter. I can't emphasise enough, especially to future UK contestants, how valuable this is.

With little more than a once-through read of everything, it's clear what everyone will score, to within a range of 1 mark. We have been strongly encouraging the use of claims and lemmas to split complicated arguments into reasonable chunks. Some of the contestants have taken this to comical extremes in places, but it's a luxury to be able to make this criticism.

Coordination and results

Gerry and I are separated by 15km, so we can't work together until the following morning. The non-geometry has been particularly uncontroversial, so we spend most of our time talking through the two sensible trigonometric arguments, and checking that the synthetic proof with reference to an inverted diagram is not a major error. As with many trigonometric arguments, there is issue that (up to additive multiples of 2π)

$$\sin x = \sin y \quad \Rightarrow \quad x = y \quad \text{OR} \quad x = \pi - y.$$

Often it is 'obvious' that only one of these options holds, but hard to give a concrete reason why, and life gets even harder with equalities like

$$\frac{\sin(x)}{\sin(\alpha - x)} = \frac{\sin(y)}{\sin(\alpha - y)}. \tag{2}$$

Renzhi has treated his instances of these with admirable (if tedious to read) care, while Thomas is relying on the bold claim that 'by the geometry of triangles, there are no other solutions' to (2). We might expect the latter to draw out the coordinators' talons, but only Michael's inverted diagram[5] turns into a point of controversy. We obtain 9 eventually, rather than the 7 which was proposed, absurdly for an argument that was elegant, direct, and entirely valid in the correct diagram up to directed angles.

Slightly earlier, the coordinators for questions 3 and 4 seem very relaxed, and we quickly get what we deserve, plus a generous extra point for Michael for using the phrase 'taxicab metric' in his rough. After the geometric diversion, Q1 is again rapid, as the coordinators say that the standard of writing is so clear that they are happy to ignore two small omissions. It transpires after discussion with, among others, the Italian leader, that such generosity may have been extended to some totally incorrect solutions, but in the final analysis, everything was fair.

So we are all sorted around 11.30am with a team score of 152, a new high for the UK at this competition. This is not necessarily a meaningful metric, but with scores of $\{20, 21, 22, 29, 30, 30\}$ everyone has solved at least two problems, and the three marks lost

[5] This had $AB > CD$ rather than $AB < CD$.

were more a matter of luck than sloppiness. Irrespective of the colours of medals this generates, Gerry and I are very pleased. We find a table in the sun, and I return to my thesis introduction while we await progress from the other countries' coordinations.

Hours pass and time starts to hang heavily, as dinner approaches, with no sign of the concluding jury meeting. Finally, we convene at 10pm to decide the boundaries. The chair of the jury reads the regulations, and implements them literally. There's a clump of contestants with three full solutions, so the boundaries are unusually compressed at 17, 30 and 32. A shame for Thomas on 29, but these things happen, and three full solutions minus a treatment of the constant case for a polynomial is still something to be happy about. Overall, 4 bronze and 2 silvers is a pleasing UK spread, and only the second time we have earned a full set of medals at this competition.

Round and about

Ceremonies

The opening ceremony took place in the middle of the problem selection process, at the students' site in Vorë. This one included a small amount of dancing and a warm speech from the deputy minister for education, and was well-judged. In particular it was brief, and the parade of teams was efficient and friendly. This year I remembered to bring some UK flags, so everyone was happy. The wholesaler had a bargain on quartered polo shirts so, unlike their flags, our team are invariant under both reflection and rotation.

The closing ceremony had a rather grander atmosphere, held in the theatre at a university for the arts in Tirana. The leaders sit in a broadly random configuration, but the teams are positioned nicely, and while we all wait there is a photo montage, featuring every possible Powerpoint transition effect, in which Jacob and his non-standard hat usage makes a cameo appearance. We are then treated to a speech by József Pelikán, who wows the crowd by switching effortlessly into Albanian, and some highly accomplished dancing, featuring both classical ballet and traditional local styles.

The lady from the ministry who is compère for the medal award section introduces some charming chaos. Leaders are called upon in large numbers to dispense the prizes, though crushingly the UK is snubbed for alphabetic reasons. There is a brief moment with forty students on stage, with twenty confused leaders, and no medals in sight. Tumbleweed rolls by, and people struggle to hold their smiles for the hordes of photographers. Eventually it is resolved, though it is a shame there is no recognition for the two contestants (from Serbia and Romania) who solved the final problem and thus achieved a hugely impressive perfect score.

The UK team look extremely pleased with themselves, and Michael's strategy to get to know all the other teams through the medium of the selfie is a storming success. The closing dinner is back in Vorë, which is very convivial and involves many stuffed vine leaves. A mass line-dance develops, where near-universal ignorance of the step pattern is no obstacle to enjoying the folk music. The DJ slowly transitions towards the more typical Year 11 disco playlist, and the adults feel 'Hips don't lie' is our cue to leave.

Tirana and Vorë

My impressions of Tirana were very positive, with long leafy boulevards, and a thriving café culture including ubiquitously excellent coffee. I have the opportunity to explore more in the interval between the exam and the arrival of the scripts. My planned trip to the Museum of Secret Surveillance is sadly foiled since it hasn't yet been opened, but there are several more statues of Skenderbeg to enjoy. The question of why he wears a goat head on his helmet remains open.

The student hotel at Vorë was highly suitable. Notwithstanding a protracted misadventure involving a camp bed, the UK students' 'suite' of rooms is excellent, and their large central room was ideal for gathering with other countries. The town itself felt like little more than a glorified motorway stop, but the side-streets branch out up into a picturesque range of hills.

I go for a climb after we have finished coordination, and find a small boy standing around selling various animals. Apparently one buys rabbits by the bucket and puppies by the barrel in Albania. On our final day we return with the students, and the animals we meet this time appear not to be for sale. Some scrabbling in the undergrowth is sadly not the longed-for bear or wolf. Many of its colleagues are loitering on the local saddle point, and our Albanian companion Elvis describes them as''sons of sheep', while Renzhi confidently identifies them as cows. They are goats, which come accompanied by a small but vigorous goatdog, who reacts with extreme displeasure to our attempt to climb to one viewpoint.

Indeed, mountains appear to be one of the main themes of Albania, with a limitless selection catering for all tastes. On our free day since have arrived early, our taste is for somewhere to practise problems that is slightly more interesting than the hotel lobby. Our guide Sebastian conjures up an excursion to Mount Dajti, a small resort two-thirds of the way up a small mountain accessed from suburban Tirana via cable car. We follow a sign that seems to point to the summit, but the trail has distinctly horizontal ambitions. We are rewarded nonetheless with some pleasant views over the mountain range down past enclosed lakes down to the Adriatic, and even beyond to Italy.

Gerry is concerned about whether our return route is actually taking us where we want to go. He is right to be concerned, but not for that reason. It is the correct direction, but through a military base. Despite this, we make it back to the top of the cable car in the correct number of pieces. There's the chance to alter this with some activities, namely horse-riding and target-shooting. The targets are balloons, mounted on a clothes line at roughly horse-head-height. Given the risk assessment's stringent take on swimming, it seems wise to give this a wide berth. Instead we visit *BunkART*, the recently-opened museum housed in the five-level 108-room bunker built into the mountain to protect Hoxha from nuclear attack. The rooms detailed the recent, fragmented history of the country, and were interspersed with aggressively modern art installations. In one basement which used to house the isotope filters, we were treated to a video loop of blood dripping onto barbed wire set to Mahler.

Beyond

The contestants had the chance to see rather more places doing their official excursions, and it sounded like the beach and the archaeological infrastructure at Dürres were particular highlights. We have our own private beach adventure on the final morning before our evening flight, with a visit to Shengjin, near the home of our guide Sebastian. From the tip of the breakwater, we see the buildings along the beachfront are a sequence of pastel colours, backing onto another sheer mountain, and we could easily be in Liguria. Jamie is revising for his A2-level physics and chemistry exams, which start at 9am tomorrow morning, and the rest of the team are trying to complete the shortlist of problems from IMO 2007. They progress through the questions in the sand, with a brief diversion to catch a crab with their bare hands for no apparent reason, and an astonishing range of fried fish.

Overall, Albania hugely exceeded my expectations and I would happily return to see more. The combination of beaches, mountains, friendly locals and cuisine based on olive oil is clearly a winning one, and I predict that the tourist Lek may become a substantial factor over the coming years.

Conclusions and thanks

All the UK students benefited greatly from the chance to attend BMO 2016, and everyone seemed to have an excellent time. We remain very glad to have been invited!

Organising a maths competition requires many people to work very hard. However, I'd like to offer particular thanks to

- Anjeza Bekolli for sorting out everything at the leaders' site. I often had no idea what I was supposed to be doing, but fortunately Anjeza a) knew; and b) didn't mind me, and pretty much everyone else too, asking about twenty times a day.

- Enkel Hysnelaj, who organised most of the academic aspects of the contest, overseeing problem selection, markscheme wording, copying and stapling $4 \times 18 \times 6$ scripts, and massaging the coordination schedule to fit those who wanted to finish early, and those who didn't. We all came for a maths exam, and without him and his colleagues, there wouldn't have been one.

- Adrian and Matilda Naco, who organised basically everything, and went far far beyond what one might expect to ensure that everyone was enjoying themselves, and their requirements were catered for. This competition had an overwhelmingly relaxed and friendly spirit, and without doubt they were responsible for this.

- Sebastian Puka, who the UK team was very fortunate to have as a guide, especially while he was preparing for exams starting the day after we left! We really appreciated his enthusiasm to show us aspects and regions of Albania, and for ensuring that even our most complicated plans proceeded seamlessly.

- Everyone behind the scenes of the UK team, including Bev from the office, and all the speakers and problem-setters at our camps and competitions, who helped so much with preparations.

- Gerry Leversha and Jill Parker, who were excellent colleagues, and our UK team consisting of Jamie, Rosie, Jacob, Michael, Thomas and Renzhi, who made many friends, solved plenty of problems during and around the competition, and were excellent ambassadors for the UK Maths Trust, and for mathematics in general.