

# 53rd International Mathematical Olympiad

UK leader's report

Mar del Plata, Argentina, 4th–16th July 2012

## Introduction

The International Mathematical Olympiad is the world's foremost mathematics competition for school-age students. About a hundred countries participate annually; each country can send a team of up to six students.

This year, the 53rd IMO was held in Mar del Plata, Buenos Aires province, Argentina. The UK has participated in every IMO since 1967. This year, the team were:

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UNK1	James Aaronson	St Paul's School, London
UNK2	Sam Cappleman-Lynes	Shebbear College, Devon
UNK3	Andrew Carlotti	Sir Roger Manwood's School, Kent
UNK4	Daniel Hu	City of London School for Boys
UNK5	Joshua Lam	The Leys School, Cambridge
UNK6	Matei Mandache	Loughborough Grammar School

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The first reserve was Adam Goucher (Netherthorpe School, Derbyshire), and the other reserves were Gabriel Gendler (Queen Elizabeth's School, Barnet, London) and Katya Richards (St Helen and St Katharine School, Oxfordshire).

I (Dr James Cranch, University of Sheffield) led the team; Jack Shotton (Imperial College London) was the deputy leader.

Dr Geoff Smith (University of Bath) attended as Observer A and in his capacity as member of the IMO Advisory Board. Bev Detoef from the UKMT's office in Leeds attended as Observer C, accompanying the team to provide pastoral support.

## Questions

As ever, this year's IMO consisted of two exam papers, sat on consecutive days. Each exam consists of three questions, and lasts four and a half hours. Each question is worth seven points.

The questions this year were as follows:

**Problem 1.** Given triangle  $ABC$  the point  $J$  is the centre of the excircle opposite the vertex  $A$ . This excircle is tangent to the side  $BC$  at  $M$ , and to the lines  $AB$  and  $AC$  at  $K$  and  $L$ , respectively. The lines  $LM$  and  $BJ$  meet at  $F$ , and the lines  $KM$  and  $CJ$  meet at  $G$ . Let  $S$  be the point of intersection of the lines  $AF$  and  $BC$ , and

let  $T$  be the point of intersection of the lines  $AG$  and  $BC$ . Prove that  $M$  is the midpoint of  $ST$ .

(The *excircle* of  $ABC$  opposite the vertex  $A$  is the circle that is tangent to the line segment  $BC$ , to the ray  $AB$  beyond  $B$ , and to the ray  $AC$  beyond  $C$ .)

(Proposed by Evangelos Psychas, Greece.)

**Problem 2.** Let  $n \geq 3$  be an integer, and let  $a_2, a_3, \dots, a_n$  be positive real numbers such that  $a_2 a_3 \cdots a_n = 1$ . Prove that

$$(1 + a_2)^2 (1 + a_3)^3 \cdots (1 + a_n)^n > n^n.$$

(Proposed by Angelo di Pasquale, Australia.)

**Problem 3.** The *liar's guessing game* is a game played between two players  $A$  and  $B$ . The rules of the game depend on two positive integers  $k$  and  $n$  which are known to both players.

At the start of the game  $A$  chooses integers  $x$  and  $N$  with  $1 \leq x \leq N$ . Player  $A$  keeps  $x$  secret, and truthfully tells  $N$  to player  $B$ . Player  $B$  now tries to obtain information about  $x$  by asking player  $A$  questions as follows: each question consists of  $B$  specifying an arbitrary set  $S$  of positive integers (possibly one specified in some previous question), and asking  $A$  whether  $x$  belongs to  $S$ . Player  $B$  may ask as many such questions as he wishes. After each question, player  $A$  must immediately answer it with yes or no, but is allowed to lie as many times as she wants; the only restriction is that, among any  $k + 1$  consecutive answers, at least one answer must be truthful.

After  $B$  has asked as many questions as he wants, he must specify a set  $X$  of at most  $n$  positive integers. If  $x$  belongs to  $X$ , then  $B$  wins; otherwise, he loses. Prove that:

1. If  $n \geq 2^k$ , then  $B$  can guarantee a win.
2. For all sufficiently large  $k$ , there exists an integer  $n \geq 1.99^k$  such that  $B$  cannot guarantee a win.

(Proposed by David Arthur, Canada.)

**Problem 4.** Find all functions  $f: \mathbb{Z} \rightarrow \mathbb{Z}$  such that, for all integers  $a, b, c$  that satisfy  $a + b + c = 0$ , the following equality holds:

$$f(a)^2 + f(b)^2 + f(c)^2 = 2f(a)f(b) + 2f(b)f(c) + 2f(c)f(a).$$

(Here  $\mathbb{Z}$  denotes the set of integers.)

(Proposed by Liam Baker, South Africa.)

**Problem 5.** Let  $ABC$  be a triangle with  $\angle BCA = 90^\circ$ , and let  $D$  be the foot of the altitude from  $C$ . Let  $X$  be a point in the interior of the segment  $CD$ . Let  $K$  be the point on the segment  $AX$  such that  $BK = BC$ . Similarly, let  $L$  be the point on the segment  $BX$  such

that  $AL = AC$ . Let  $M$  be the point of intersection of  $AL$  and  $BK$ . Show that  $MK = ML$ .

*(Proposed by Josef Tkadlec, Czech Republic.)*

**Problem 6.** Find all positive integers  $n$  for which there exist non-negative integers  $a_1, a_2, \dots, a_n$  such that

$$\frac{1}{2^{a_1}} + \frac{1}{2^{a_2}} + \dots + \frac{1}{2^{a_n}} = \frac{1}{3^{a_1}} + \frac{2}{3^{a_2}} + \dots + \frac{n}{3^{a_n}}.$$

*(Proposed by Dušan Đukić, Serbia.)*

## Results

Here are the results of the UK team:

	name	Q1	Q2	Q3	Q4	Q5	Q6	total	award
	James Aaronson	7	0	4	7	2	0	20	Bronze medal
	Sam Cappleman-Lynes	7	0	0	7	0	0	14	Bronze medal
	Andrew Carlotti	7	7	5	6	0	4	29	Gold medal
	Daniel Hu	7	0	0	7	2	0	16	Bronze medal
	Joshua Lam	7	1	0	7	7	0	22	Silver medal
	Matei Mandache	1	4	0	7	0	2	14	Bronze medal

The medal boundaries were 14 for a bronze medal, 21 for a silver medal, and 28 for a gold medal. Hence everyone on our team obtained a medal, which is a fine achievement. In addition, we were joint first in Western Europe (jointly with the Netherlands, who have had an extraordinary year). However, we placed only fourth in the Commonwealth of Nations: we were beaten by Canada, Singapore and India, all of whom deserve congratulations.

For completeness, Table 1 shows the scores and ranking positions of the top thirty-three countries, and various others whose performance has traditionally been of interest to us.

Clearly, this has been an excellent year for South Korea. Another noteworthy international performance was by Teodor von Burg of Serbia: he now moves to the top of the IMO Hall of Fame with a bronze, a silver and four gold medals from the last six IMOs.

Moving on to discussion of our own team, Andrew is still on course to be the most successful British IMO contestant of all time, ranked by medals obtained: to do this he will need a gold medal at IMO 2013 in Colombia, which will be no easy task.

James will be disappointed that his uncharacteristically modest performance has left him with a bronze medal, but this is due to his extraordinarily high personal standards: we are as satisfied with him as with the rest of our team.

Next year, Andrew, Daniel and Matei are all available to compete again, as are reserves Gabriel and Katya. We greatly look forward to their contributions.

On the other hand, James, Sam, Josh and first reserve Adam are all off to Trinity College, Cambridge: we are sure that they will continue to excel there.

country	total	rank
Republic of Korea	209	1
People's Republic of China	195	2
United States of America	194	3
Russian Federation	177	4
Canada	159	5
Thailand	159	5
Singapore	154	7
Islamic Republic of Iran	151	8
Vietnam	148	9
Romania	144	10
India	136	11
Democratic People's Republic of Korea	128	12
Turkey	128	12
Taiwan	127	14
Serbia	126	15
Peru	125	16
Japan	121	17
Poland	119	18
Brazil	116	19
Bulgaria	116	19
Ukraine	116	19
Netherlands	115	22
<i>United Kingdom</i>	115	22
Belarus	114	24
Croatia	110	25
Greece	107	26
Australia	106	27
Hong Kong	106	27
Saudi Arabia	105	29
Republic of Moldova	104	30
Germany	102	31
Israel	102	31
Mexico	102	31
		⋮
Belgium	93	38
France	93	38
Hungary	93	38
Italy	93	38
		⋮
New Zealand	75	53
		⋮
Ireland	34	78
		⋮

Table 1: The top thirty-three and selected other countries in IMO 2012

## Leader's diary

Many of the events this year were overshadowed by the untimely death of Sam Cappleman-Lynes's mother, at the beginning of the IMO. While, clearly, it would be inappropriate to include these events in my narrative, the reader should understand the difficulties involved. We were all impressed by Sam's determination in successfully participating under such conditions.

### Sunday 1st July

London's Piccadilly line takes me to Heathrow Terminal 5, and I am only ten minutes later than the appointed meeting time. There I meet the team, our pastoral assistant Bev, and James's parents who have agreed to take delivery of the uniform.

Jack Shotton arrives half an hour late, but we have left ourselves a vast amount of spare time and he has hardly dented it.

We check in, and decide to eat in the Giraffe restaurant the other side of security. The waitresses, from their vantage point a respectful distance away, admire Sam's absent-minded solving of Rubik's Cubes (and a range of similar beasts, one of them large and dodecahedral).

Then we embark on the journey, allegedly the single longest-distance flight offered by British Airways. I am kept company by Andrew and Josh.

I glance out of the window from time to time, and ascertain that our cruising altitude is roughly two metres, and that the mid-Atlantic ocean looks a lot like a big piece of sheet aluminium.

Blankets and pillows are provided, and we make good use of them: not much happens.

### Monday 2nd July

We arrive at Buenos Aires Ministro Pistarini airport. Baggage reclaim is a very slow process, but that gives me enough time to disappear and wash my face and brush my teeth. After doing that I feel almost like a human being again.

We are then met by a minibus driver and taken to St George's College, Quilmes, a traditional British international school south of Buenos Aires.

We are met by their head of maths, Stephen Kay. He shows the students to the boarding accommodation, then welcomes Jack and me warmly to his flat, introduces his wife who gives us tea and freshly-baked Canadian snacks, and allows us to shower and change. He explains how he has rearranged the timetable in order to provide us with unfettered use of a classroom. In gratitude, Jack and I immediately conspire to allow his dog to escape, causing him to spend fifteen minutes rounding it up again.

After lunch we try to find old IMO shortlist problems our students have not done, and talk through them. The Argentineans eat late and, at this school, have afternoon tea at 5pm.

We play frisbee (except for Bev, who does not indulge) in the evening, but it becomes dark rather suddenly soon after 6pm. Soon afterwards the Australians arrive. It is nice to see their leader Angelo di Pasquale and deputy Ivan Guo again. This year, Angelo has brought his wife Hellen: he missed the IMO last

year in order to marry her; clearly he has waited until this year in order to take her on honeymoon.

## Tuesday 3rd July

We get up feeling refreshed, and at breakfast time we familiarise ourselves with *dulce de leche*, a milky caramel substance which is omnipresent in sweet Argentine food.

The students sit the first practice paper, during which I give a short master-class on continued fractions to a handful of school students. They speak English very nicely, and are a joy to teach.

After lunch we mark the paper. The British students have performed nearly uniformly well: all have done the same two questions, and nobody has done the hard third question. We debrief the students on the paper.

Before dinner, I give the students what I naively imagine to be a rousing pep talk, and present important logistical advice. Mostly this consists of reminding them that Jack and I have to argue with non-native English speakers about the strengths of their writings, and encouraging them not to indulge in partying until the point where Paper 2 is a recent memory.

In the evening there is a party for us staff at the headmaster's residence, which renders my advice to the students ironic.

## Wednesday 4th July

After breakfast, I bid the students *au revoir*, then embark on the journey south. A taxi takes me from the school to Buenos Aires's bus station.

At the station I see Arturas, the Lithuanian leader: we strike up conversation, and it turns out we will be sharing a bus. We become confused together about the buses: our bus is not listed on any of the boards. With a few minutes to spare, we strike up conversation with a nearby family, who explain that they are in the same position. One of them goes to ask, and discovers that the bus is late, and that late buses are not listed on the boards until they have arrived.

In the end it arrives. The bus turns out to be extremely comfortable: my seat is, in essence, a bed. Some kind of ghastly Eddie Murphy movie is played, but it is easy to ignore, particularly as Eddie Murphy spends most of the movie in silence.

Arturas and I arrive in Mar del Plata, and are met by smiling IMO staff. We are beckoned into a taxi, which drives us about 10km south to a countryside hotel. There we meet many old friends.

I have a nice room, but it is extremely warm. I spend a while trying to locate the source of the heat so that I can neutralise it. Eventually I realise that there is underfloor heating over which I have no control. So I simply open the window and resolve to avoid the floor.

I pick up the shortlist. By tradition, a shortlist contains seven or eight problems in each of four areas: algebra, combinatorics, geometry and number theory. The organisers list them in order of perceived difficulty. So C7 might be expected to be a very hard combinatorics problem, and G2 might be a much easier geometry problem.

After some time enjoying the shortlist, I decide to go and be sociable. I walk back to the hotel bar: representatives of Germany, the Netherlands, and New

Zealand are there with Matjaž, the IMO's technical support man. So I sit with them for a while.

### **Thursday 5th July**

After a brief welcome meeting, where a few logistical points are discussed, we have a free day to consider the shortlist.

After getting hopelessly stuck on one of the combinatorics shortlist problems, I notice there is a little shop in the hotel which sells bric-a-brac including replacements for forgotten swimming shorts, and I pick up a pair.

Upon going to the pool I discover that the sizes, which I had taken to be classified by waist size in Imperial inches, in fact follow some other system, and that I have bought a very large pair of shorts. No wonder the girls in the shop were laughing at me. I deploy ingenuity of a sort that I lacked when attempting the combinatorics problem, and find a way of tying one part of the shorts to another part of the shorts. I now simply have a fashionably baggy pair of shorts, and can go swimming.

### **Friday 6th July**

The day begins with the first proper jury meeting. The first item of business is the task of discarding questions which are already known, or which are uncomfortably close to known questions.

One leader begins by standing up and suggesting that problem A2 bears some resemblance to a problem on IMO 2000. The chair of the jury nods sagely, and the problem is struck off. There is an uproar, where various members of the jury explain that it would be normal to have a formal motion, translate it into the other official languages of the jury, and then vote on it. The chair of the jury indicates that he is happy for us to do so, "if we want to spend all day voting". He pushes it to a vote, and does not permit discussion, though we have not even been shown the original question to which A2 is being compared. Naturally, many people wish to abstain, but the chair of the jury is uninterested in having abstentions counted. "Why do you want to play these games?", he asks.

After this crash course, the chair of the jury rapidly comes up to speed with standard procedure, and happily we are able to scrutinise all further removals. This year, the IMO question gods will be pleased with their sacrifice: a total of six questions are burnt.

### **Saturday 7th July**

While yesterday felt unproductive, it turns out that sleeping on everything has led to us having a good understanding of the problems. Today's debate is largely well-informed.

The jury start by picking two easy questions. There are two obvious candidates, A1 and G1, and those are rapidly chosen.

Then we have a difficult fight over the two hard problems. There is a number theory problem, N7, which has clearly won the jury's hearts, but it is not obvious whether its partner will be geometry or combinatorics. Eventually, after much talking, two geometry problems are rejected (including G5 which is much-liked

but slightly too easy to be a hard problem) leaving the combinatorics question C6 standing.

During this process, Geoff walks in and is greeted by many old friends.

After lunch, one leader suggests it might be possible to save ourselves time: he proposes that we vote to select the recently-rejected geometry problem G5 straight away. So we vote on whether to vote on this. The results are 38 all, with seven abstentions. The large number of abstentions leads me to think we should have voted whether to vote on whether to vote this problem onto the paper first.

The mechanism of selecting questions is traditional. First people propose individual problems, until the jury tire of this. Then people propose pairs of problems, until the jury tire of this. Then, finally, there is a voting procedure, following a devil-take-the-hindmost system over several rounds. The numerical details are, by tradition, agreed on an *ad hoc* basis every time, but it turns out surprisingly often that we agree to eliminate  $\lfloor \sqrt{n} \rfloor$  pairs of problems every time there are  $n$  remaining.

The medium questions are eventually selected. They do indeed include G5, and also the algebra question A3.

It is time for the English Language committee to meet: we have two hours to reformulate the problems, taking into account any inadequacies or ambiguities, and hopefully making it easier for translation into the fifty-four other languages in which the papers will be sat. It seems to have become traditional for the UK leader to chair this committee (and I tend to invite the New Zealand leader Chris Tuffley to be “secretary”, which, among other things, involves doing all the work). By the time we are done persuading the jury of the merits of our work, and incorporating their several excellent suggestions, I am quite tired. Accordingly I spend a relaxing evening annoying the Francophones by sitting in on their translation meeting with a beer.

In the evening some good news reaches us: the results of our annual pre-IMO contest against the Australians, the Ashes of Mathematics. The UK has won it by the comfortable margin of 72 points to 52, corresponding to three more problems solved. Geoff is understandably unhappy at having to break the news to Angelo.

## Sunday 8th July

Today is a big day. This is not least because it is the first time a British man has been in a Wimbledon men’s singles final since 1938. However, naturally enough, our man Andy Murray loses to the rampaging Roger Federer. The Swiss leader Julian Kellerhals acts magnanimously enough about the whole matter.

Informed partly by sleeping on it, and partly by other people’s problems in translation, Chris decides to further polish the English. There are few complaints.

Now the English version is done, we are left with little to do for the day, while many other language versions are produced.

## Monday 9th July

Today is Argentina’s Independence day: the 196th anniversary of their declaration of independence in 1816. The organisers have timed the IMO so that the

opening ceremony will fall on this auspicious day.

We are put on buses to a concert hall in Mar del Plata: we are met by loud drumming coming from a crowd of carnival performers.

The opening ceremony is blissfully free of politicians: the only people who speak are mathematicians, and the speeches are short. We start by standing for the Argentine national anthem, which breaks into rock music in places.

The IMO anthem, a gift to the world from the first Argentine IMO in 1997, is played by three accordionists and a singer.

The parade of nations occurs: every country marches across the stage in turn. Our team look very smart in embroidered blazers. For the first time, an IMO oath of fair play is sworn by all contestants.

Then two big cannons fire vast amounts of confetti into the hall. When the air clears, we observe a man come onto stage equipped with two small hammers on metre-long cords, one in each hand. When these are whirled at speed, he becomes a tapdancing quadruped.

After being taken back to our base, the rest of the day is spent on mark schemes. The coordinators have been hard at work devising mark schemes, and they seem to be sensible (after a few corrections suggested by the jury). My idea of a perfect mark scheme is one which provides some structural insight into a problem. These mostly aren't like that, but they do at least look like they might work.

One difficulty this year is that they have had to construct mark schemes in such a way as to not reveal alternative solutions to problems: this is a novel security measure devised by the advisory board.

The other piece of business in this long evening's work is the elections for the IMO advisory board. First is the election for secretary, to replace John Webb of South Africa. There are two candidates, Indra Haraksingh of Trinidad and Tobago, and Gregor Dolinar of Slovenia; the latter wins. In a moment of high drama, Radu Gologan of Romania ties with Rafael Sanchez of Venezuela for the post of general member, with 45 votes each, and then concedes to him. While this act of kindness was conspicuous, I am proud to be in a community where elections are conducted in such a friendly manner.

## Tuesday 10th July

Today is the day of the first IMO paper.

It is traditional for students to be allowed to ask questions in the first half-hour of a paper, in case they do not understand the problems. In recent years, the jury and the students have been far apart during the paper, and questions have been relayed electronically. This year, however, there is less than 20km separating us, and instead of transporting electrons from the exam site to the leaders' site, we will be transporting leaders from the leaders' site to the exam site. This is a feature that invites nostalgia from the more experienced leaders.

Our students ask no questions, which makes us happy. We had feared that the complicated wording of Q3 would create huge problems, but things turned out fine in the end.

After the end of the Q&A session, we stroll downstairs and out of the building, finding ourselves immediately amidst a crowd of deputy leaders trying to board some buses. Since I am not supposed to communicate with the deputies until after Paper 2, this may be regarded as a bit of a security breach. Jack

is nowhere to be seen. The Canadian deputy waves at me from a bus, smiling mysteriously, but then I realise with embarrassment that the Canadian leader Jacob Tsimerman is right behind me. I see one pair, a leader and a deputy, stroll down the street as the old friends that they are.

After we are safely put on separate buses, we are taken off to a barbecue in a country estate. There is a quick tour of the vineyards and fields, and then we are taken into a large tiled barn with several log fires. There are *empanadas* (traditional Argentine Cornish pasties), sausages and black pudding, grilled chicken and grilled steak.

We are kept entertained by some musicians, and later by some tango dancers and a singer with a large collection of costumes made from a small amount of fabric. Then some drummers come out, and the aforementioned quadrupedal tapdancer. He uses his hammer-on-a-rope to knock a cigarette out of the New Zealand leader's mouth. It's a filthy habit, and I'm sure it dissuades him from adopting it.

After lots of food and several glasses of wine, and with the scripts from day 1 looming later in the day, it seems sensible to join a group who plan to walk back to our hotel. It turns out to be a very pleasant walk of about 10km, across the coastal lowlands.

The scripts arrive, and I spend the rest of the evening assessing them.

### **Wednesday 11th July**

Today is the second IMO paper. Today, again, our students make us proud by not asking any daft questions (or indeed any questions at all).

One question says that a certain named nearby student keeps making loud noises with his hands, and asks if he can be made to shut up. We decide, amidst some amusement, that the answer must be "yes", and send orders to threaten him appropriately.

After this we move into a hotel across the road from the students. I am reunited with Jack at once, who brings me news of our students. Apparently, Sam's habitual Rubik's Cube solving led to him being mobbed by admiring schoolgirls at St George's. Also, the team members have developed a love-hate relationship with *alfajores*, the Argentine snack obtained by inserting a layer of dulce de leche between two sweet biscuits, and coating the whole thing in chocolate.

Afterwards I meet the students coming out of the exam. They feel battered by the difficulty of the paper, but perk up after realising most other countries feel similarly.

Jack has a hard day: not only must he get up to speed on the scripts from Day 1, we must both get used to the scripts from Day 2. We spend a long evening in, reading them.

### **Thursday 12th July**

Jack and I take an opportunity to visit the students' recreation room, where they are based when nothing else is happening.

I enter, and find that Sam is engaged in a chess tournament. A few yards away, Andrew Carlotti has joined an international effort to make progress on a 24000-piece jigsaw puzzle the size of a small carpet. Daniel is learning circus

skills, and can juggle hoops for short periods of time. The others are all milling around.

The room is excellent: besides these activities there are all manner of board and table games, table football, pool, video games, ping-pong, people teaching tango dancing and acrobatics, and at some hours, karaoke.

Rumours reach us of some unfortunate hijinks the night before. Apparently a certain team had a raucous party in their rooms, with various sorts of contraband present, and some damage to property. Options with that team were mulled over, but in the end the organisers felt they had no choice but to rusticate them, sending them to a hotel the other side of town.

A stop-and-search policy is instituted, and Jack and I have our bags inspected every time we cross the road to enter the students' hotel.

After lunch, we begin coordination, the process of agreeing scores with committees of locals. This starts with problem 6. We were asking for three points for Andrew and two for Matei.

The coordinators say that Andrew's work might even be enough to receive four points, but that they wish to think about it some more in comparison with other scripts. We're in no hurry, so this seems an entirely adult approach.

In the evening, Jack and I feel well in control of all the scripts, so we pop out to a pub in town, in the company of Mark from Ireland and Marteinn from Iceland. The first appropriate venue we reach is in fact the local Irish pub. The atmosphere and clientele are pleasant and there is a local stout and a brown ale on sale.

Perhaps the only significant area where the pub is seriously deficient is that of the local drunk. This pub's drunk, a man with the implausible name of Dariak, besieges us, tells us repeatedly that he is both an artist and is well-connected, and refuses to believe that Jack is not German. His outgoings on business cards must be substantial, for it seems giving us one each is probably not enough. Eventually two nearby girls, Vicky and Camila, take pity on us and magic him away; we are grateful for the change of company.

## Friday 13th July

Today the students will be making friends with dolphins at the local aquarium, while Jack and I do the bulk of the coordination.

It starts with good news on Problem 6: the coordinators have decided that Andrew's script (and all similar scripts) deserve the extra mark mentioned earlier.

However, Andrew loses a mark on Problem 4. This is entirely self-inflicted: he has a daft strategy for checking one of his solutions, and tells the reader he can't be bothered to carry it through to a conclusion.

We also have a fairly painless time on Problem 3, and pick up 5 for Carlotti and 4 for Aaronson, both of whom manage all of the first part and make inroads into the second.

Problem 2 is rather less straightforward. Five scripts are uncontroversial, but the clash is over Matei's. We had planned to ask for two points, but the coordinators seem intent upon giving it six. Matei has proved the equality for all  $n \geq 11$  (and we have checked that the argument works equally well for  $n = 10$ ). However, the given bound becomes pathetically weak as  $n$  becomes large, and, as Jack and I have spent a long evening verifying, the method can't easily be

adapted for the cases  $4 \leq n \leq 9$ . So, in our opinion, Matei has no programme for solving the full problem.

Jack and I ask permission to go talk in a corner. We are agreed that this mark is inappropriate, and that Matei would prefer to receive an honourable mention (the standard award for a student with a full solution to some problem, but who receives no medal) than a decidedly dishonourable bronze.

The coordinators offer four, since it's the arithmetic mean of what we are asking and what they are asking. We explain that that is no good. Indeed, our absolute demands are that the student gets a mark which is both fair and in line with those given to other students; this would achieve neither.

When we return this afternoon, having decided to deliver an ultimatum, things are made uncomfortable for us: the organisers have decided to impose no hierarchy on the coordinators this year. Ordinarily there would be a chief coordinator responsible for all coordination, and problem captains, one per problem. The former post is in fact specified by the organisers' own contest regulations. As a result, we have no option but to reason with all coordinators simultaneously, and we are aware we have no recourse beyond that other than to the full jury.

With twelve coordinators sitting in a semicircle around us, we ask that they either reconsider the mark for Matei's script and for all scripts with similar work (we later discover there are five of them), or else we will be forced to bring the matter to the jury. They concede, and go to work on remarking. Later, we meet them again. They make some sensible points in favour of Matei's script – that it is closer to a solution than he is aware – and Matei ends up with a much more sensible mark of four.

Problem 1 is hard work, but of a less extraordinary sort. Five of the scripts are uncontroversial solutions. Jack has found a mark in Matei's script: in one place there is a mysterious formula. Jack has determined that it is a slightly incorrect statement of Menelaus's theorem, as applied to a certain triangle involving a novel constructed point. Jack has also found a full solution which starts with (a corrected version of) the same formula. The coordinator takes ten minutes to reproduce these formulae herself, and then agrees that this is worth a mark. This pushes Matei one step closer to an honest bronze.

## Saturday 14th July

Coordination finishes for us rapidly with Problem 5. Josh has done it, and there are points to be found for James and Daniel. The coordinators congratulate us: apparently Josh's script is the only script they have seen which succeeds by synthetic means.

This leaves us with 115 points. We are pleased with this: it is clear already that some strong countries have been damaged by this very difficult pair of papers, and we are heartened that the UK has weathered the storm.

I decide to seat myself in the hotel lobby, and gossip with all who pass. I find this just as good as sitting in front of the scores screens around the corner.

There are considerable rumblings about Problem 4: many people apparently feeling that the coordinators have interpreted the markscheme in an unacceptably strict fashion.

The Canadian staff invite me to join them in the swimming pool. I spend an hour and a half with them, and with Mark from Ireland. In the hot tub,

meanwhile, one leader well known for such things is harassing a female deputy leader: it is all rather sad.

Jack and I also spend some time in the students' hotel. It is Sam's birthday, and we congratulate him.

We also are in time to see James Aaronson win the IMO *Ta-Te-Ti-To* tournament (this game is four-in-a-row, played by two players on a  $4 \times 4 \times 4$  cubical board). John Papantoniou from Australia has reached the final of the chess contest, but we are not around to see him beaten by a Lithuanian student.

Then I return to the hotel lobby. Mea Bombardelli, the Croatian leader, has signed off with 110 points in total, which this year is enough to beat Germany. She is intrigued that her team always seem to get about 110, no matter how hard the papers are.

## Sunday 15th July

Bev comes over to the leaders' hotel for a cup of coffee, in advance of the final jury meeting. After Jack and I have done with breakfast we head down. There is a lengthy delay as the organisers work out how to project things on a screen.

One leader is good enough to open the proceedings by telling the other assembled leaders how coordination works: clearly he feels that the point of it was lost on some of them.

It turns out that there are three scripts left unresolved, all on Problem 4, and it is the job of the jury to settle them. We vote first on two Estonian scripts, and in the end we uphold the suggestions of the coordinators.

Then Zuming Feng, from the USA, gets to discuss his unresolved script. He suggests that the organisation of the coordinators is the worst he has seen in fifteen years or so: the coordinators wish to penalise his student partly for verifying a symmetric two-variable homogeneous fourth-degree polynomial identity by eye, rather than by multiplying out both sides. This is not the International Multiplication Olympiad, he thunders.

The coordinators want to give it five. The script looks very similar to our student Andrew's six, so I am comfortable with a mark of six for it. This is what happens, eventually, but there are procedural misgivings all around.

A leader from Eastern Europe proposes that we change the definition of "honourable mention" this year, to award honourable mentions for students with fives and sixes in Problem 4. I find the suggestion amusing but unconstructive.

Then there is a short break, and we go into the more normal business of the final jury meeting. There is a report from the chief invigilator, and we approve the scores and medal boundaries. Two new IMO hosts are announced: following the already-known hosts for the next three years (2013 in Colombia, 2014 in South Africa and 2015 in Thailand) will be IMO 2016 in Hong Kong and IMO 2017 in Brazil.

We take the team out for lunch at a restaurant down the road. There is steak and pasta, all very pleasant. The students tell us that Andrew spent the evening before partying until 6am and then spent the time between then and breakfast solving last year's shortlist problem C8 (the "napkins" problem).

I give the students their scripts back, and award the *golden pen* for most painful script which nevertheless receives a full score to Andrew, for his work on Problem 2. On balance, this year has been more characterised by the team

setting about the problems like so many angry ferrets, and obtaining large numbers of marks by bravely and repeatedly trying to do the problems, but sadly we have no prize for that. I also award the *Sceptre of UNK*, my IMO jury voting stick, to Matei Mandache. It will endow him with demigodlike powers for the next year.

We hurry from lunch to the closing ceremony. There is more of the national anthem, more of the IMO anthem, and the medals are awarded. Jeck Lim of Singapore, who achieved the unique perfect score, is ushered on stage alone to receive his gold medal.

After this we relax for a while, and drink a bottle of rosé champagne that Josh Lam's father has donated. The students discuss their principal source of intrigue: a female student from an Asian country phoned their rooms overnight, asking if she could speak to UNK4 (Daniel). However, events during the daytime suggest that she miscalculated and that it was UNK5 (Josh) that has attracted her interest. None of the team have any idea what to do about this.

Then there is the closing party, taking place in a large hall in the students' hotel. Our team are ushered to a table, and we join the Pakistani leader Barbu Berceanu, an expatriate Romanian. Barbu appears to enjoy the chance to talk to our Romanian speaker Matei.

The food arrives eventually, and then entertainment is laid on from the other end. First up is the annual award of the *golden microphone*, to the leader who has made the most jury speeches. This year there has been a clear winner: the jury has been pleased to listen most often to the pronunciations of Lev Radzivilovsky of Israel. This gives way rapidly to the finals of the *IMO's Got Talent* competition. Highlights include some pleasant singing by a Romanian student, and a bizarre routine by two Pakistani students, one of whom plays the piano while performing contortionistic moves, while the other spins a plate on a stick.

I retire early: the next two days will be hard work.

## Monday 16th July

The day begins with a five-hour bus and half-hour taxi ride back to Buenos Aires's main airport. The film this time is a lengthy melodrama by Steven Spielberg about a horse. By the time the film is halfway through, I am simultaneously yearning for the early days of Spielberg, and also hoping that the next character to appear will be a glue-boiler or a Kazakh butcher or someone else with a similarly robust attitude towards horses.

We are, because of scheduling issues, left waiting for several hours in Buenos Aires airport. One eagerly-anticipated source of excitement was waiting for Bev to check Daniel Hu in, as for some unknown reason the website didn't let her do so last night. However, she has her boxing gloves on and so this takes just fifteen minutes.

While she is remonstrating with the staff of Iberia, we are mobbed by several crowds of Argentine teenage girls, who are apparently all driven wild by British accents. Presumably, since chiselled Latin men with designer stubble and impressive dance moves are two-a-penny around here, what women around here really hunger for is clumsy ill-dressed pasty types from the UK.

We decide to hide from the adoring hordes in an airport café. There we order a large amount of pizza to tide us over. James, who doesn't like food with

ingredients in, gets a small pizza *sin queso*.

We mostly engage in playing liar's poker, but Bev reports that at the other end of the airport is a big display about the Falklands war. I go to have a look. Every banner of photos is entitled, "las Malvinas son Argentinas y sus recursos naturales también" ("the Falklands are Argentinean and so are their natural resources"). There are lots of pictures of Argentine soldiers in successful moments, and infographics about sunken British warships. I was startled by this as an introduction to the country for visiting foreigners. However, I am pleased to realise that our dealings with individual Argentine people over the course of this trip have in no way reflected the unease between our governments.

The queues for security and emigration passport control are lengthy; the whole process takes a while, and I refresh myself with a fizzy pomelo juice drink afterwards.

Eventually we get on a plane. Sam and Andrew, next to me, fall asleep immediately, which makes for a very quiet flight.

## Tuesday 17th July

We all wake up, and calculate that Sam has spent seventeen of the last twenty-six hours sleeping. Before too long we arrive in Madrid Barajas airport, only about 50 minutes late. The landing is not exactly smooth; many people experience reverse peristalsis on the way in.

The delay has caused us to miss our connecting flight; the staff juggle us all on the next flight to Heathrow, and give us one-third as many meal vouchers as there are people in the party.

To get to the next flight, we have to go to the very other end of the airport. Madrid is very, very hot. I become stressed, and Jack beats some sense into me.

There is some confusion and delay in working out where in the departure hall the restaurant *ARS* is, since this is the unique restaurant at which the meal vouchers are redeemable. Eventually we determine that the *ARS* is to be found at the rear end. They sell us sandwiches, crisps and drinks to go and we catch our flight home. Soon afterwards we arrive at Heathrow, tired and smelly, to be met by four friendly parents.

## Acknowledgements

It is no easy matter to send six young people seven thousand miles, or to prepare them for competition at the highest level against young people from other countries, or to support them before and during this process.

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*James Cranch, 19th July 2012*