

# **Balkan Mathematical Olympiad 2005**

## **Iași, Romania, May 4 - 10**

### **Report of the UK team**

Report by Robin Bhattacharyya, Leader of the UK team

#### **Introduction**

The United Kingdom has entered the International Mathematical Olympiad (IMO) every year for nearly forty years now. There are surprisingly many other international competitions for mathematically talented school students, organised on a smaller and more regional basis. One of them, which has run since 1984, is the Balkan Mathematical Olympiad (which I will refer to as the BalkMO), which is for countries in South-East Europe. In the last couple of years the member countries of the BalkMO have invited one or two other teams to participate, and this year the UK was asked for the first time.

In the UK the British Mathematical Olympiad (BMO) is an examination taken by very bright school students; there are two rounds, with about one hundred students invited to take the second round, as a result of their scores in the first round. The UK selected its team of six for the BalkMO in February, on the basis of the scores in the BMO. In fact the selectors of the team were already familiar with many of these students from training camps which they had attended after getting high marks in contests in previous years.

An important point to make is that to give more people a chance to experience an Olympiad, we deliberately did not select the four people at that time thought most likely to be IMO team members; in fact all four went on to achieve at least a silver medal in IMO 2005. As far as I can tell, most of the other teams did send their strongest teams. Certainly the twenty-four participants at the IMO from Romania, Moldova, Turkey and Albania consisted of twenty-three who had been at the BalkMO, plus one other who had been selected for the BalkMO but who could not make it. We finished 11<sup>th</sup> out of 14 teams in Iași, while of the competing countries only Romania, Hungary and Bulgaria finished ahead of us in the IMO in Mexico later in 2005.

#### **The Team**

The team chosen in February, with ages in brackets, was :

Konrad Dąbrowski	(17)	(Clitheroe Royal Grammar School)
Chris Kerr	(18)	(Winchester College)
Edward Newkirk	(18)	(an international school)
Jack Shotton	(15)	(Portsmouth Grammar School)
Lee Zhao	(16)	(Nottingham High School)
Bo Zheng	(17)	(Glasgow Academy)

The team leader was Robin Bhattacharyya (a teacher at Highgate School) and the deputy leader was Amanda Turner (a graduate student at Cambridge University).

## **Departure**

I have to admit that my knowledge of the city of Iași was very sketchy back in February, but we discovered that it has more than 300,000 people, is a former capital city of Romania and is culturally as important as anywhere in the country. Iași airport however must be one of the quieter ones in the world, with just two flights coming in each day, one from Bucharest and one from Timișoara. The Bucharest flight seated only 48 people so we were relieved to find that we could book for eight people on the right day. At least two teams were too late in booking, and had to take a six or seven hour train journey instead, after flying to Bucharest. Other teams were even slower, arriving by bus from their home countries, or in one case travelling by a combination of transport taking over two and a half days !

We set out early on Wednesday morning from Heathrow, which meant some very early starts – about 4.30am for Lee, but he wasn't too tired to chat very enthusiastically while we waited at the airport. Only Amanda had been to Romania before, and she hadn't visited Iași, so it felt like quite an adventure for all of us. We would be meeting people from many countries that we would not normally ever meet someone from.

We had a few hours to kill at Bucharest's airport, where we met the Hungarian team who were waiting for the same flight on to Iași. Many of our team already knew members of the Hungarian team from December (2004) when there had been a joint training camp in Hungary for Olympiad squad members from both countries. Unfortunately someone had brought some playing cards, and a game of Mao ensued among the students from both teams, despite disapproval from the UK leadership for that particular game.

We got to Iași very late at night, and while the team and Amanda went to their hotel to find dinner ready for them, I went straight into a Jury Meeting at my hotel, where the leaders were given the 17 problems that had been shortlisted for the tournament, as well as some very useful maps and other information about the Olympiad. We were to be given the solutions to the problems the following morning, which gave us a bit of time to try to work through some of them. There were some very nice problems that various countries had submitted, with a spread of problems from the four main areas of Olympiad question mathematics – geometry, number theory, algebra and combinatorics.

I was delighted to find dinner set out in my very pleasant hotel room when I eventually got there, after midnight, and I had a go at some of the questions while enjoying the view of Iași from my tenth floor window, before getting some much needed sleep.

## **Opening Ceremony**

The opening ceremony was held in a splendid auditorium in the university in Iași, with the mayor of Iași and other local dignitaries present; local TV was there too. It was very pleasing to see how proud the city was to host the event and how genuinely happy the speakers were that Iași had been chosen. Romania held the first ever IMO in 1959 and they have been very enthusiastic (and successful) Olympiad participants since then. There were banners advertising the Olympiad hanging above roads around the city, and even the tram drivers knew that this mathematics competition was taking place.

I was on a balcony with the other leaders, while the students, who had of course not yet seen the shortlisted problems, were below. Straight after the ceremony the leaders were taken by coach to an unknown location – so unknown that I still have no idea where we went, except that it was in a nice big house somewhere in the middle of agricultural land just over half an hour's drive from the city.

Meanwhile, the students were treated to a reception at the university, followed by a tour of the mathematics library and the university's private planetarium. Afterwards, their guide, Corina, showed them around Negruzzi High School where she was an English teacher and where the exam would be sat the following day. Everyone was extremely impressed by the marble staircases and the notice boards detailing the achievements of pupils and teachers. The rest of the day was spent on a brief tour of the city and an early night in preparation for the following day.

## **Leaders Select the Questions**

The participating teams in the Olympiad were :

(BalkMO members)

Albania  
Bulgaria  
Cyprus  
Greece  
Macedonia (Former Yugoslav Republic of)  
Moldova  
Romania  
Serbia-Montenegro  
Turkey

(Invited teams)

Hungary  
Kazakhstan  
Romania 2<sup>nd</sup> team  
United Kingdom  
Yakutia

The leaders of the nine countries present that were member countries of the BalkMO were allowed to vote in Jury Meetings, while the leaders of the invited teams, including the UK, were not allowed to vote, but all the leaders would have their views heard. (Incidentally there is another member country of the BalkMO, Bosnia-Herzegovina, but that country tends not to send a team to the BalkMO).

Mircea Becheanu, a former leader of the Romanian IMO team, chaired the Jury with excellent common sense and calmness and in a good spirit. Radu Gologan (leader of the Romanian IMO team, but here as the chief organiser) and other members of the Problem Selection Committee, who had put the shortlist of questions together from problems submitted by the participating nations, were also available to help in discussions about the questions. Mircea and Radu both have experience at IMOs in organising / problem selection / mark co-ordinating, quite apart from their experience as IMO leaders, so the competition was in very good hands.

We talked about which problems we felt were the right ones to select, with some formal voting at times. The Hungarian leader Jozsef Pelikan, a very big name in the IMO world, as Chair of the IMO Advisory Board, had many useful things to say about the problems, and other leaders put their ideas forward, some more than others. Problems were classified as 'easy', 'medium' or 'hard' (of course none of them is entirely easy !) by the Problem Selection Committee. The aim is to achieve a balance of levels of difficulty in the question paper so that the students can be well separated. There was some reclassifying, based on the leaders' judgements of the questions. There was also discussion of alternative versions of some of the questions, to make them slightly easier or slightly harder, in an attempt to set a paper at the right level. The Bulgarian leader for one was keen that the paper not be too easy.

In the end we went for one question from each area, once Jozsef Pelikan and others had become convinced that the combinatorics questions were not just number theory in disguise. As the paper has to stand as a whole, instead of picking one question from each area in isolation, we discussed different possible sets of four questions to make up the paper. However the hardest decision turned out to be finding the most suitable from several possible algebra questions, and eventually a certain inequality question was chosen ahead of a rival functional equation question. Two of the four questions had been altered from the shortlist versions, both times to make them slightly harder.

I was asked on a couple of occasions about the wording of questions. The Jury Meetings were conducted in English, even though there were no native English speakers apart from me, which was quite strange for me. It wasn't so great for the lady from Yakutia who spoke no English at all ! Yakutia, also known as Sakha, is a very large but very scarcely populated area of Siberia, not an independent country, but an organiser of an international maths competition itself. There were other Russian speakers there, and quite a bit of quiet interpreting was going on during the Jury Meetings.

In the afternoon there were hours of translating from the English version into the ten other official languages of the competition (Albanian, Bulgarian, Greek, Hungarian, Kazakh, Macedonian, Romanian, Russian, Serbian and Turkish). It is considered important that the translations are as close to the English original, word for word, as is possible, and great care is taken over the translation. Obviously there wasn't much for

me to do at this time, but I could enjoy the sun and the view of the peaceful countryside and farm land around, and I chatted to some of the Romanian committee that had prepared the shortlist. When the eleven versions of the question paper had been printed in their various scripts and were sat next to each other on a table, they were quite a sight.

With the problem setting complete, that evening we had a kind of barbecue, with two whole lambs roasted outside over a fire. There was plenty of Romanian wine and spirit (țuică) flowing, and the music system was turned up, first playing Right Said Fred, the Supremes and other non-contemporary pop music on Romanian radio, and then blasting out Deep Purple, Bob Dylan and the Beatles from CD / tape until the early hours. I talked quite a bit with the Romanians on the Problem Selection Committee, and I discovered that some of them remembered John Rickard (the UK's most successful IMO contestant ever in terms of gold medals and special prizes) from the mid-1970's and they held him in awe, and were very sad to hear that he had passed away. Without BBC World and CNN, which I had in my hotel room but not at this house, I was blissfully unaware of what was happening on election night in the UK.

### The Questions

The following morning (Friday) we had the competition itself. The students had four and a half hours in which to tackle the following four questions as well as they could.

- 1) Let ABC be an acute-angled triangle whose inscribed circle touches AB and AC at D and E respectively. Let X and Y be the points of intersection of the bisectors of the angles C and B with DE and let Z be the midpoint of BC. Prove that the triangle XYZ is equilateral if and only if angle A is equal to 60 degrees.
- 2) Find all primes  $p$  such that  $p^2 - p + 1$  is a perfect cube.
- 3) Let  $a, b, c$  be positive real numbers. Prove the inequality  $(a^2/b) + (b^2/c) + (c^2/a) \geq a + b + c + 4(a-b)^2/(a+b+c)$   
When does equality occur ?
- 4) Let  $n \geq 2$  be an integer. Let  $S$  be a subset of  $\{1, 2, \dots, n\}$  such that  $S$  neither contains two elements one of which divides the other, nor contains two elements which are coprime. What is the maximal possible number of elements of such a set  $S$  ?

(Note :  $p^2$  means “p squared” and  $\geq$  means “greater than or equal to”)

A coach took the leaders back on Friday morning into Iași so that we could rule on any queries about the wording of the problems that the contestants might submit in the first half hour of the event. There were only two questions, one of which was from the UK, and which I proposed to answer with the word ‘Yes’ (Chris wanted to know whether question 4 asked for some kind of formula involving  $n$ ). This went to a vote and was accepted. The high school where the students took the exam was very grand (there was a lot of polished marble / stone). The leaders were there for a bit longer

discussing the mark schemes that the Problem Selection Committee had prepared – in particular we decided that merely noting the equality case  $a = b = c$  in question 3 was worth 0 marks out of 10, instead of 1 mark out of 10, against the wishes of some of the leaders who wanted there to be a ‘social mark’. There was also an idea among the leaders that for some of the questions the mark scheme assumed just one way of solving the problem, while the students might be able to think of another way, and that more flexibility in the mark scheme might be a good idea.

## **Iași**

We finally got a chance to see some of Iași later that morning, while the students were still in the exam rooms, and during the afternoon. Iași is the capital of Moldavia, which is one of the three main provinces of Romania, and not to be confused with the independent country of Moldova. We had a reception hosted by the deputy mayor of Iași in which he told us about the city. In terms of history and culture Iași can rival anywhere in Romania. The public buildings were very impressive – the university, the high schools, the theatre, the library, the museums (some of which are to be found in one magnificent building) and the many churches, not least the Metropolitan Cathedral, Romania’s largest Orthodox church. The apartment blocks where most people in the city seemed to live were less inspiring concrete buildings of several storeys, but many of the roads were lined with trees, which added some colour.

Those leaders who wanted to go went to the Botanical Gardens, which were lovely in the sunshine. The students would not be so lucky when they went on a later day in the rain ! We were guided by a local English teacher, Alina, who took the three of us who were young and keen enough on a walk down to the lake, through much of the Gardens. Ali from Turkey, who it transpired shared a friend of mine from his time in the US, and Irena from Macedonia, who could talk perfectly well with the Serbian and Bulgarian leaders even when each spoke only their own language, were the other two leaders there. It was all a very nice contrast to the grey of some of the city.

The students meanwhile were entertained by a display of folk dancing and some, Lee in particular, demonstrated considerable talent in the area! In fact, dancing seemed to feature quite heavily on the itinerary, with organised discos every night. Chris rose to the challenge, earning himself quite a reputation among the participants. The rest of the team decided this wasn’t really their scene, however, and chose to play cards back at the hotel on subsequent evenings.

## **The Marking**

The students’ answers were photocopied during Friday afternoon and the photocopies went to the Romanian co-ordinators for the marking, while Amanda and I got the originals for our team to scrutinise in preparation for our discussions with the markers on the following day to decide the final marks. Amanda, whom I hadn’t talked to since arriving in Iași on Wednesday evening, made the fifteen minute walk up the main avenue of the city from her hotel to mine, and we set about studying the scripts of our team members very closely. We started at 7pm and didn’t finish until 3am. Each of the four questions is marked out of ten, so even if someone has essentially solved the problem there are many places in which marks might be lost, and some marks might be available even if a student has clearly not solved the whole problem.

Amanda had spoken with our team (the deputy leaders spent much more time with the teams than the leaders) and they thought that two of them had solved the geometry question, one of them had managed the number theory problem, one had solved the combinatorics question, but none had made progress with the tricky algebraic inequality. We worked out that this was indeed the case - it was a relief to find that no-one who thought they'd got something out then found out that they hadn't - and we spent a lot of time reading certain scripts, in particular Edward's correct but incomplete solution to the combinatorics question. By the time we had finished we had a very clear idea of exactly what useful ideas and progress each member of our team had made on each question, and we were ready for the co-ordinating of marks (and some sleep first!).

On Saturday, Amanda and I met the co-ordinators to agree the marks, back at the Negruzzi High School where the exam had taken place. The co-ordinators, Romanian university mathematicians, had also read the scripts very carefully and we had to justify our claims for marks in detail in several places. The combinatorics question seemed to me from the scores for other countries that were going up on the notice boards to have been marked strictly for all countries (there were quite a few scores slightly below 10, but not many perfect 10's), and Amanda and the co-ordinators spent quite some time discussing individual lines in Edward's work before we came to a compromise and agreed the mark.

Between them our contestants had solved three of the four problems, but no one of them had managed more than one problem, which was slightly disappointing. Our marks were as follows :

	Q1	Q2	Q3	Q4	Total
Konrad Dąbrowski	3	2	0	0	5
Chris Kerr	0	10	1	1	12
Edward Newkirk	0	1	0	8	9
Jack Shotton	10	1	0	0	11
Lee Zhao	0	1	0	4	5
Bo Zheng	10	2	0	0	12

(Each question was marked out of 10)

It was a very rainy day, but as the mark co-ordinating finished earlier than expected for our team, and as the team were being entertained elsewhere, Amanda and I had a chance to do a little bit of sightseeing ourselves and we headed off for a brief trip over the River Prut.

With the marking for all teams completed the grade boundaries were set. The aim is that about two thirds of the students from member countries get medals, and the medals are distributed among them in the ratio 1 : 2 : 3 for gold : silver : bronze. A total score of 10 was needed for a bronze medal, so Chris, Jack and Bo earned bronze medals, while Edward unfortunately missed by a mark (Amanda and I had pushed hard for a mark of 9 on Question 4, but there was no way that the co-ordinators were going to give us the extra mark).

## **Excursions**

That night (Saturday) the leaders had a reception at the National College High School. A girl at the school showed us around. The walls were covered with results of National College students in various regional and national academic competitions (in mathematics and in many other subjects). These things are taken very seriously in Romania, and a good performance will automatically get a student into the university of their choice (quite different from the UK situation). The school was another very grand one in terms of the building, corridors, etc.

Sunday brought the main excursion of the Olympiad, to the stunning painted monastery of Voroneţ, where a nun told us about the brilliant paintings on the outside walls, which had been produced over 450 years ago. Voroneţ is in Moldavia, but a couple of hours away from Iaşi. We also visited the impressive Citadel in the town of Suceava. This was even older, dating from when Suceava was the capital of an independent Moldavia, from the 14<sup>th</sup> to the 16<sup>th</sup> century, before Iaşi took over. The third part of our excursion was to an oceanographic museum / aquarium at Botoşani.

The team were fairly despondent that morning as although they had been sent their marks by text, one of the other participants had worked out the bronze medal cut-off to be 13 and they were under the mistaken impression that they had all missed out! They were soon reassured, however, and throughout the rest of the day everyone was in good spirits.

## **Farewell Dinner**

We had a chance to let our hair down at the Farewell Dinner that night. Even before the excellent food had been finished, and the cake was brought out in a circle of fire on the floor, the music system was in operation, and young (and not so young) mathematicians from around Europe were to be found on the dance floor. Edward was early to bed, but the rest of the UK delegation were to be seen dancing. I don't think that I'll forget the usually serious looking Moldovan leader's enthusiastic version of the Twist, or the IMO's main man Jozsef Pelikan eventually getting up to dance to a Peter Andre song. One of the Albanian delegation was taking quite an interest in Amanda on the dance floor. What nobody who was there will forget was Chris Kerr's incredibly energetic and keen display, going on without faltering right up until 3am when everything shut down. Chris, with his hundreds of photos, and his determination to meet every student and every team's local guide during the week, was quite a character at the Olympiad and one of the organisers was suggesting that he be given a special medal for diplomacy !

## **Closing Ceremony**

Monday morning brought the closing ceremony in the spectacular National Theatre. A youth orchestra played some Brahms, and there was also Romanian folk singing and dancing, again by school age people. The medals were awarded on stage. Romania had won the team competition by some way; they were the only country to come to terms with the difficult inequality question. Bulgaria were second, and Moldova confirmed recent Olympiad strength by coming third. Chris, Jack and Bo collected



their medals. Only one student, Adrian Zahariuc of Romania, managed to score full marks in the competition, while another Romanian dropped just one mark, and a third Romanian lost only two marks.

The final results by team were :

1) Romania	189
2) Bulgaria	146
3) Moldova	139
4) Kazakhstan	134
5) Romania II	128
6) Turkey	127
7) Hungary	115
8) Greece	99
8) Serbia-Montenegro	99
10) Albania	71
11) UK	54
12) Macedonia (FYR)	48
13) Yakutia (team of 4)	30
14) Cyprus	23

I had met our team's guide Corina on the trip to Voroneţ. She is a teacher in Iaşi, and it was interesting for me to talk with her about teaching in Romania; the experience seems more similar to teaching in the UK than I had imagined, with very similar emotional rewards and frustrations with behaviour. Corina told us that the local TV channel would be showing a half hour documentary about the Olympiad that evening. After a brief visit to a plush shopping mall, the UK team watched the programme together, and Corina translated for us. Jack, Chris, Lee and many organisers and other students were shown being interviewed about their experiences, and highlights of the Olympiad activities were shown, including the Botanic Gardens for the students in heavy rain.

After a special dinner for the leaders at a restaurant serving traditional Romanian food, and alcohol, it was (fairly) early to bed to get ready for a very early flight back to Bucharest and then to London. This proved to be quite uneventful, except for one of Amanda's many bottles of Romanian wine getting crushed on the plane in the baggage hold, soaking some of her things.

### **Final words**

The competition was organised superbly, the hotels were very comfortable, I thoroughly enjoyed the food (and good provision was made for those on our team who ate a restricted range of food) and we were shown some fine parts of Iaşi and Moldavia. Good questions were set for the competition paper; the inequality was certainly hard (only 17 of the 82 students present managed more than 2 marks out of 10), while the other questions were found to be less daunting, but not trivial. I enjoyed meeting the leaders from the other countries, and I ended the week in good spirits, as did everybody in our team.

The performance of the UK team in Iași was not nearly as strong as it has been in the IMO in recent years. Obviously we didn't send our strongest possible team; in fact Jack was the only one from the Balkan Olympiad to go on to be part of the eight in the squad for the UK team to the 2005 IMO. The IMO team also receives some extra training, at Oundle School and then in the week before the IMO, that the BalkMO team did not have.

I would like to thank first our team – Konrad, Chris, Edward, Jack, Lee and Bo you all represented your country well. Our guide Corina told me how impressed she was with all of you. Amanda did an excellent job of looking after the team and of getting every last mark out of the co-ordination that we could have got. Thank you also to all our schools, especially my own, Highgate School, who allowed us to take time out in term time to attend the Olympiad; and of course thank you to Geoff Smith, Adrian Sanders and everyone else who has helped to mentor these students, mark their BMO exams and teach them at various training camps over the years.

I finish with some words from Jack Shotton about the Balkan Mathematical Olympiad:

“it ... was a valuable experience, and a brilliant opportunity to see Romania and meet some like-minded people, even if only briefly.”

Report by Robin Bhattacharyya, UK leader at the Balkan Mathematical Olympiad in Iași, 2005 (with some parts written by Amanda Turner, UK deputy leader in Iași)