40TH INTERNATIONAL MATHEMATICAL OLYMPIAD

BUCHAREST, ROMANIA

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Report by Imre Leader (UK Team Leader)

This report is about the 40th International Mathematical Olympiad, which was held in Romania in July 1999. The IMO is the pinnacle of excellence in mathematics for school pupils the world over. Each year, around 80 countries each send a team of 6 contestants to the IMO. There they sit two $4\frac{1}{2}$-hour exams, each containing just three questions. Medals are awarded for good performances. This year the host country was Romania, which organised the first two IMOs (in 1959 and 1960), as well as the IMOs of 1969 and 1978 (the expert reader will wonder why the 1999 IMO is number 40, instead of 41 – it is because there was no IMO in 1980).

Let us start with some of the events leading up to the IMO. The selection of the team started with the Senior Mathematics Challenge, a multiple-choice paper sat by more than 40000 students, taken in November 1998. The SMC lasts 90 minutes, and consists of 25 questions, of which the first 15 are meant to be widely accessible and the last 10 rather more testing. Based on their performance in the SMC, around 1000 contestants proceed to the next round, the BMO1. This is a far, far harder paper, containing just 5 questions to be done in $3\frac{1}{2}$ hours. Anyone who solves a BMO1 question has reason to feel pleased with himself/herself! The BMO1 is held in mid-January, and is followed by an amusing weekend in which 20 or so academics, teachers and ex-olympians gather together to mark the scripts.

After BMO1, about 100 pupils qualify for BMO2, which is a still harder exam, consisting of 4 questions to be attempted in $3\frac{1}{2}$ hours. Based on BMO2, 20 pupils are selected for the Trinity Training Session at Easter – these 20 include those who we feel are realistic

1

contenders for the team, and also some younger pupils who we believe are good prospects for the future.

The Trinity Training Session is an extremely intense and exciting experience for everyone. It lasts four days. For the first three days, the students have a variety of sessions, some taught to the whole group of 20 and some taught in groups of 6 or 7. The emphasis is on the students trying problems: the actual amount of 'lecturing' is kept to a bare minimum. The final day is by contrast rather different. The main event is the last of the selection exams, the Final Selection Test. The FST is designed to resemble a real IMO paper: there are just 3 questions, and the time allowed is $4\frac{1}{2}$ hours.

In the next day or so, a squad of 8 is selected. The choice is based on performance in FST, BMO2 and BMO1, and also on how the students have performed during the Training Session. The 8 are notified within a few days of leaving Trinity, and they then embark upon the final and most gruelling part of the selection. This is the dreaded Correspondence Course. Each week to 10 days, the students are sent a sheet of about 8 hard problems. They send in their solutions, which are marked by the Leader and the Deputy Leader (both new this year: I took over from Adam McBride as Leader, while Richard Atkins, Head of Maths at Oundle, took over as Deputy from Philip Coggins). After about 5 rounds of this, the team of 6 is chosen, with the other 2 acting as reserves. Of course, the two reserves contribute immeasurably to the success of the team, as their presence during the training course has forced people to work hard for their places in the team!

In the week before the IMO itself, the team gather at Birmingham, where the Summer School for younger pupils is held. As well as participating in some of the events of the Summer School, the team receive some final training and preparation.

This year, the squad of 8 was as follows.

Team:

Thomas Barnet-Lamb (Westminster School)

Rebecca Palmer (Clitheroe Royal Grammar School)

Marcus Roper (Northgate High School)

Oliver Thomas (Winchester School)

Oliver Wicker (Cockermouth School)

Jeremy Young (Nottingham High School)

Reserves:

Stephen Brooks (Abingdon School)

Michael Spencer (Lawnswood High School)

Of these 8, Jeremy was the only 'returner' from last year's IMO in Taiwan (where he won a Bronze medal). Rebecca was one of last year's reserves.

Next came the IMO itself. The IMO, for the students, was to start in Bucharest on July 13th, but the Team Leaders flew in three days early, to select the questions that would be used. Each country has the right to submit some questions (months in advance); the host country then narrow these down to a short-list of about 25 questions, and it is from these that the six must be chosen. For those three days, the Leaders were kept in a secret location, far from Bucharest – in fact, they were allowed no contact at all with the teams or the Deputy Leaders until the last exam had finished, for obvious reasons! This year, the Leaders met in Poiana Brasov, a ski resort high in the Carpathians (Dracula territory).

Some countries send an 'Observer' with the Leader or Deputy: this is usually someone who will do the job in a later year, and is coming along to see how things work. This year, I had Adam McBride as an Observer with me. Needless to say, this was not for him to see how things worked, but to show me!

The Jury chose the questions, and supervised the various translations. This year, 81 countries participated, which necessitated more than 50 languages. The Jury consists of all the 81 Leaders: as one can imagine, a committee of this size is a rather curious beast. Once the exam was ready, the Leaders travelled to Bucharest, but were kept well away from where the teams were staying.

Meanwhile, the team, led by Richard Atkins, had arrived in Bucharest. After acclimatising, and a long Opening Ceremony, the actual exam dates were July 16th and 17th. There then followed a period of 48 hours of intense activity by the Leaders and Deputies. The Deputies move to the Leaders' hotel, to help with the marking of the exams. Each country marks its own students' scripts, and then goes to 'coordination' for each question: this involves meeting with two Romanian mathematicians and agreeing on marks. Finally, totals are worked out, and the cutoffs for medals established. The rough principle is that

the ratio of Gold to Silver to Bronze to no medal should be very close to 1 to 2 to 3 to 6.

After a few days of sightseeing and socialising, there was a Closing Ceremony, at which the medals were awarded. Everyone flew home the next day.

Now on to the papers. Each day has three problems, to be done in $4\frac{1}{2}$ hours, with each question worth 7 points.

FIRST DAY

Problem 1. Determine all finite sets S of at least three points in the plane which satisfy the following condition: for any two distinct points A and B in S, the perpendicular bisector of the line segment AB is an axis of symmetry for S.

Problem 2. Let n be a fixed integer, with $n \geq 2$.

(a) Determine the least constant C such that the inequality

$$\sum_{1 \leq i < j \leq n} x_i x_j (x_i^2 + x_j^2) \ \leq \ C \Big(\sum_{1 \leq i \leq n} x_i \Big)^4$$

holds for all real numbers $x_1, \ldots, x_n \geq 0$.

(b) For this constant C, determine when equality holds.

Problem 3. Consider an $n \times n$ square board, where n is a fixed even positive integer. The board is divided into n^2 unit squares. We say that two different squares on the board are *adjacent* if they have a common side. N unit squares on the board are marked in such a way that every square (marked or unmarked) on the board is adjacent to at least one marked square. Determine the smallest possible value of N.

SECOND DAY

Problem 4. Determine all pairs (n, p) of positive integers such that p is prime, $n \leq 2p$ and $(p-1)^n + 1$ is divisible by n^{p-1}.

Problem 5. Two circles Γ_1 and Γ_2 are contained inside the circle Γ, and are tangent to Γ at the distinct points M and N, respectively. Γ_1 passes through the centre of Γ_2. The

4

line passing through the two points of intersection of Γ_1 and Γ_2 meets Γ at A and B. The lines MA and MB meet Γ_1 at C and D, respectively. Prove that CD is tangent to Γ_2.

Problem 6. Determine all functions f from the reals to the reals such that

$$f(x - f(y)) = f(f(y)) + xf(y) + f(x) - 1$$

for all real x and y.

You are invited to send in solutions, enclosing an SAE please, to:
Imre Leader, Dept. of Mathematics, University College London, Gower St., London WC1E 6BT.

The countries of origin of these questions were Estonia, Poland, Belarus, Armenia, Russia and Japan, respectively. Although none of the United Kingdom's problems were included, a remarkable three of them made the short-list: one by our veteran problem-setter David Monk (Edinburgh University), and one each by recent ex-olympians Ben Green and Mansur Boase (both now at Cambridge). Mansur's problem was actually very nearly chosen.

The team came away with an impressive haul of medals: three Silver and two Bronze. As a whole, the UK team's total score of 100 points (out of a possible 252) put us 20th of the 81 countries. When looking at such results, it is important to bear in mind that our training programme is tiny compared with those of other countries (most of which have training camps lasting one or several months, and in some cases even a whole year!). Indeed, perhaps a more revealing statistic is that, among the Western European countries (where long training camps are the exception rather than the rule), we came 2nd, beaten only by Germany. Here are the top 20 teams, with their scores:

182 China and Russia, 177 Vietnam, 173 Romania, 170 Bulgaria, 167 Belarus, 164 Korea, 159 Iran, 153 Taiwan, 150 USA, 147 Hungary, 136 Ukraine, 135 Japan, 130 Yugoslavia, 116 Australia, 109 Turkey, 108 Germany, 107 India, 104 Poland, 100 UK.

Here are our individual scores, with marks per question:

	Q1	Q2	Q3	Q4	Q5	Q6	Total	
Thomas Barnet-Lamb	7	5	1	4	1	1	19	Silver
Rebecca Palmer	4	0	6	3	2	1	16	Bronze
Marcus Roper	6	1	0	1	5	1	14	Bronze
Oliver Thomas	5	6	1	3	5	1	21	Silver
Oliver Wicker	1	2	1	3	2	1	10	
Jeremy Young	7	2	2	7	1	1	20	Silver

The top mark was 39, obtained by three contestants. Rebecca was delighted to work out that her mark made her the 5th best girl in the world. Oliver Wicker was rather unlucky to miss out on a Bronze medal by just two marks: the cutoff for Bronze was 12 points. The Silver cutoff was 19, while that for Gold was 28.

The IMO was a magical experience for the Leaders as well as for the Teams. For the Leaders, there was the wonderful initial 3 days of meeting as the Jury, all thinking and arguing about the problems and discussing different ways to solve the questions, then the unbelievably hard two days of marking and coordination, and then the last few days socialising with the team and other teams. For the contestants, there was the rather awe-inspiring arrival, being plunged among the best few hundred school pupils in mathematics from around the world, then the tension of the actual exams, then the sometimes equally great tension of waiting for the results, and in amongst all this the meeting with other teams from around the world.

Perhaps the best way to convey some of the atmosphere of the IMO is to record my daily diary of some of the things that happened.

July 10: Adam and I arrive at the main Bucharest airport, Otopeni, after a 4-hour flight from Heathrow. We are met by the guide for the UK team (each team is assigned a Romanian guide, usually a language student) and his girlfriend, the guide for the Icelandic team. We are told we will be going on a drive to Poiana Brasov, about 3 hours away – up until that moment, we have had no idea where the Jury will be meeting. But first we have to wait for the Cuban Leader, as he will be in the car with us. We wait and wait. Finally, someone does appear, saying 'I am the Polish Leader'. Consternation for our hosts, who had forgotten when he was coming. The Cuban Leader arrives, and off we all go, crammed into a car. The Polish Leader has been to Romania before, and is an expert on what

6

will happen. He says 'In an hour and a half, we will be in the mountains, with incredible views', but he also tells us that in an hour and a quarter it will be dark. He is right on both counts.

The car ride is terrifying, with lots of overtaking around blind corners. We get to the hotel at midnight, completely exhausted. Hoping for some dinner, we wander into the (obviously closed) restaurant, to be told 'we are closed'. We look so miserable that the waiter tells us that he will see if he can find us a sandwich. We sit down and are given cold meat and salami. Then it turns out that this is only the start of the meal! We are given a huge, delicious dinner. We get to bed much happier.

July 11: After breakfast, the list of shortlisted problems is given out (without solutions!). We have the whole day to go back to our rooms and think about the problems. It is a wonderful sensation – there are so many beautiful problems that it is hard to know where to start. It is pure pleasure.

At 6pm our first Jury meeting (apart from a brief organisational session that morning) is scheduled to take place. The Leaders sit alphabetically by country, with the Observers (who can watch but not speak or vote) at the back. Luckily, UNK is very near the end of the alphabet, so Adam is sitting right behind me, and can whisper words of wisdom to me and swap banter about some of the sometimes rather odd things said. Not much happens, except that a problem that has been used in one country's training programme is removed from the list. Also, solutions to the shortlisted problems are given out.

Just before and after the Jury meeting, several Leaders wander around the room, handing out presents to the other Leaders. I accumulate about 30 presents over the next few days, ranging from coffee from Columbia to a leather wallet from Iran. Plus green playing cards from the Irish, port from Portugal, rum from Trinidad, and many other excellent things. Most Leaders also give out copies of their national competition booklet, and the UK in turn give out about 200 copies of the BMO booklet (kindly carried to Romania by the team!).

July 12: Jury meetings all day. The chairman (from Romania) bravely tries to keep control of 81 people. We have a succession of votes: typically, there is some discussion

(all in English), and then, just before voting, the motion is translated into Russian and Spanish. We vote by raising our 'wands' – like large plastic toblerone bars. Things are pretty chaotic. We decide to have a 'beauty contest': each person will vote for his six or so favourite problems, so that a profile can be built up on the front-runners among the short-list. We spend ages deciding how many problems each person will be allowed to vote for. But there are dafter things to come. For we are also going to have a difficulty poll (each person saying how hard he think each problem is, so that a large chart can be displayed), and now the discussion turns to which of these two things should happen first. People make impassioned speeches about how each vote might affect the other. Finally, after half an hour, we take a vote on the question of in what order we should hold the beauty contest and the difficulty poll. I forget what the result was.

July 13: More Jury meetings. At some points, events move faster than the chairman can follow. For example, after someone has proposed a motion, and demanded a vote straight away, the Hungarian Leader makes a speech about how, in Jury meetings, it is traditional to always allow a period of discussion about a motion, rather than quickly voting on it. He sits down, and the chairman says 'OK, so now we vote'.

We start by trying to choose two 'easy' problems (of course, easy is a relative word – as anyone who has ever tried to do an IMO problem will realise. But the rough aim is to have, by IMO standards, two easy questions, two medium ones, and two hard ones). Again there are some complications waiting to happen. It is clear that there are about 6 problems that could be classified as 'easy' (one of these is definitely not easy, but some of the Jury have been fooled into thinking it is easy by the fact that is has a *short* solution – which of course is *no* guarantee that a question is easy). So how should we select two? We decide to vote by choosing our favourite easy problems, and eliminating the least popular. But how many problems should each person vote for? And how many should be eliminated each round? Then another suggestion appears: we should list the *pairs* of easy problems that seem good (this is clearly sensible, as for example one would not want two problems on the same topic). We vote to do this, and must then again go through the business of deciding how many pairs each person will be allowed to vote for each round, and how many of the least popular pairs will be eliminated each round. Someone points out that,

8

according to what we have just voted for, each Leader will be allowed to vote for two pairs, even when we are down to just two pairs left! More fine tuning occurs.

Just as we are about to get voting, someone asks what will happen if, say, three pairs are equal last at a stage when we going to eliminate just two pairs. He is shouted down with cries of 'pedantry' and 'we should get on with voting'. Needless to say, three pairs do indeed come equal last! Even the recording of results of votes is complicated. What happens is that one person counts the votes on the left side of the room and another counts the votes on the right side. They shout the numbers to the chairman, who announces the total for the person manning the overhead-projector to put on the screen. This sounds perfect, but in most votes there was some Leader who misheard one of the numbers being shouted, and so thought that the chairman had added up incorrectly. The chairman was actually right every time.

In case this sounds like chaos, it is worth pointing out that, in almost every case, the Jury *do* come to a sensible conclusion. It is just that the manner of convergence to such a conclusion can be convoluted!

We now take an afternoon break, and some of us (Adam, myself, the Polish Leader and the Irish Leader and Observer) decide to walk up the hillside. The view is spectacular – steep slopes of pine trees stretching above and below us as far as the eye can see. The others decide to carry on higher, but I am too lazy and tired to climb further.

Now comes voting on two difficult problems, and then two medium problems. The venerable Hungarian Leader speaks strongly in favour of Mansur's problem, but it does not get chosen. At the final vote, I face a horrible dilemma. We are down to just two possibilities for the last question to be chosen: an inequality and a number theory question. I am absolutely sure the UK team will do better on the inequality than on the number theory, but unfortunately I feel that the number theory question is the nicer question (more elegant, more appealing). After some soul searching, I vote for the number theory question. To my annoyance, and also to my relief, the inequality is chosen. (Footnote: in fact, the team end up not doing very well on the inequality after all!)

Finally, we have chosen our 6 questions – a nice, interesting and rather hard paper. But the day's work is not done, for now comes the preparation of the English version – the

official version, on which all translations will be based, so that it is important that all of the wording is as clear and precise as possible. The six or seven English-as-first-language Leaders gather round a table, but about 20 other Leaders hang around, giving 'helpful' suggestions and generally slowing things down enormously. Finally, the job is done, and the Australian Leader presents our version to the Jury. He does a great job of dealing with hecklers, and the version is approved within a few minutes. Adam notices that the spelling 'center' appears, and wants me to protest, but I am scared to be labelled a pedant in my first time as Leader, so I do not say a word. I am teased about this by nearly every English-speaking Leader.

July 14: The five official translations (French, German, Spanish, Russian, Chinese) are produced and approved. Then the full set of more than 50 translations is produced (of course, the English-speaking team leaders have very little to do at this point), and laid out for everyone to see. The Latvian and Lithuanian Leaders explain that their languages do not have the symbol '$\sum$', so they ask permission to write out the sums in Problem 2 is expanded form. After a huge amount of debate, the ludicrous solution that we vote for is to give *every* student in the whole competition an extra piece of paper, headed 'Clarification', spelling out what the sigma sign means. Even here, there is extra complication: we spend a few minutes trying to decide whether it should be an 'explanation' or a 'clarification', decide on 'clarification', and proceed to the vote on whether or not this sheet is to be given out. After the vote, the New Zealand Leader, who is a native Russian, points out that the vote was invalid, because when the Russian Leader translated the motion into Russian he used the word for 'explanation' instead of 'clarification'! He is shouted down.

I get back to my hotel room and suddenly remember that tomorrow is my 10th wedding anniversary – one loses all track of time during the IMO. A hasty call is made to Interflora.

July 15: This is the day for our transfer to Bucharest. Off we go, in three coaches, with a police escort down the winding mountain road. The driver of our coach turns off the air conditioning (it is incredibly hot and humid), and later puts on the radio. Eventually, the Hungarian Leader makes his way to the front of the bus, and tells the driver 'We are willing to swap the radio for some air conditioning'. Apart from stopping for the Brazilian

Leader to be sick, not much else happens, and we arrive in Bucharest after three hours. Our hotel is even more palatial than the one in Poiana Brasov had been.

Then it is off to the Opening Ceremony. We are kept on a gallery up above, to avoid any contact with the contestants, but we manage to find and wave to the team and Richard. The ceremony starts with a good national anthem (the first national anthem I have ever heard which actually has a good tune), followed by huge numbers of speeches, all along the standard lines of 'maths is future of world...new millennium...many young people...meeting of minds...make friends from other countries'. Eventually, it is all over, and we wave good luck to the team.

July 16: Competition day. We start with our first breakfast in our new hotel, which turns out to be rather wonderful. A buffet stretches over about eight tables, consisting of bacon, sausages, eggs (scrambled, fried), cold meats and salamis of every conceivable sort, rollmops, other kinds of fish, three kinds of melon including watermelon, cereal, croissants, rolls, bread full of strange seeds, breadsticks, danish pastries, jam doughnuts, apple strudel, cheesecake and, on one occasion, chocolate cake as well.

After breakfast, the Jury are bussed to where the exam will be taking place, and escorted into a room, with many guards making sure we do not wander off anywhere near where the students are. Here we are all set to answer questions. For the first half hour, the contestants are allowed to ask questions. Each must be written down, and sent to the Jury. The relevant Leader reads out the question, and the Jury argue about what the reply should be (usually, it is 'read the Problem again', or 'no comment', but sometimes of course we are helpful, as when someone asks what a certain word means). Once the Jury are agreed, the Leader writes down the answer, which is then conveyed back to the student. The questions range from the idiotic ('what is my contestant code?', when each participant has had this on his name badge since arrival) to the sensible. Much hilarity amongst the Jury for some of the questions. One contestant asks, about Problem 1, 'must the points of S be distinct?' – we have no idea what he means, but decide that 'yes' must be the correct answer.

We are then taken to an open-air museum of village life, mainly to make sure we are far away from the contestants when they come out at the end of the exam. Then back to

the hotel for a late lunch. We wait eagerly for the scripts to arrive. Every single script must be photocopied first, so that the coordinators can have a look in advance, and this is an immense task, involving about 20000 sheets of paper. So we are told that we will only be getting the scripts around midnight. We wait around, and I discover that the Irish Observer knows an astonishing number of jokes, although not too many suitable for polite company. The Polish Leader goes for a midnight swim in the outdoor pool. Finally, at about 12.45, the huge bundles of scripts are brought in. Suddenly, Leaders materialise from every corner of the hotel, all keen to see how their students have done.

I settle down to have a look, and the first impression is not good. It looks like a complete disaster – little do I know that most Leaders are thinking exactly the same thing, as the exam has been found even harder than most of us thought. The Israeli Leader is almost in tears.

I stay up all night to mark the scripts, with my only company after 3am being the Irish Observer, who is spending the whole night looking at just one script (the student concerned ended up with a Bronze medal, with exactly 12 marks, so clearly this work was not in vain). I look at Problem 2 first, as I am sure the team will all have done it. But this is not the case. Oliver Thomas (referred to as Ollie for the rest of this report, to distinguish between the two Olivers) has solved it, neatly and elegantly. But he has completely forgotten about the case of equality! On to Thomas' attempt. He has done the case $n = 2$, and then found a nice reduction from the general case to the case $n = 2$. Problem solved. Well, except that, by a silly slip, he *thinks* he has reduced only to the case $n = 3$. So he has spent an hour and a half writing pages and pages, trying in vain to plug a gap, when the gap is not even there! (It is worth pointing out that mistakes such as these are very understandable, given the immense stress the contestants are under. They have been building up to this exam for weeks or months, and in addition they feel that they are 'flying the flag'). No-one else has done the question. I am deeply unhappy.

On to the very difficult Problem 3. Only Rebecca has solved it. But her solution is utterly wonderful and very non-standard (there is a tiny gap in her solution, but it is easily filled). It is a joy to read – I am happy again.

Now for Problem 1, where it seems that everyone has written pages and pages. Jeremy

has done it, with a very clearly-written solution. Thomas has done it (with a small and irrelevant gap). Rebecca and Ollie have done a bit, and Marcus has a long solution that is full of good ideas but contains an error.

July 17th: Second competition day. Again we have our question-and-answer session. Then it is time for the Senior Coordinators to tell us their proposed mark-scheme. Each question has 6 Coordinators (who work in three teams of two), plus one Senior Coordinator who flits between the pairs during coordination. These Senior Coordinators are clearly very wise and sensible. Each one tells us the mark-scheme he will use for his question. Then the Jury have a chance to comment (although, at this stage, they cannot *order* the Coordinators to do anything). The Senior Coordinators listen politely, agreeing with the sensible comments and saying things like 'I will think about it' for the non-sensible ones.

We then hang around until the exam has finished, and now at last we are allowed to see the teams. All six of the team say the exam has been terrible, 'even worse than yesterday'. Adam and I follow Richard and the team to their place for lunch, and then Richard transfers, from the non-palatial student accommodation to the rather nicer hotel the Leaders have been staying in.

That evening, the scripts arrive, and the three of us start to pore over them. No-one has got anywhere with Problem 6. Jeremy has done Problem 4, but nobody else has. Oliver has been unfortunate here: his speciality is number theory, and I would expect him to solve a question like this 99 times out of 100, but he has only got halfway through. There are some other part-solutions. Meanwhile, Richard and Adam are struggling to make sense of what has been written about Problem 5. It seems that Marcus, although not having solved the question, has produced all the ingredients – he has just not noticed that if he puts them together the right way he will have a solution. Ollie has also made some progress. Not much else. We are full of gloom – it looks as though we are heading for one Bronze (Jeremy) and nothing else. But then we go for a walk down to the hotel foyer, and find many other Leaders and Deputies there. We soon find out that the papers have been found hard universally – this puts us in slightly better spirits.

July 18: The first coordination day. We have been given our time-slots for our half-hour

13

sessions with the Coordinators. The first one for us is Problem 2. The three of us gather in the ante-chamber just outside the huge room where coordination takes place. I am incredibly nervous, with butterflies in my stomach – in fact, the same feeling as one has before an exam. Then we are called in. The room is subdivided into 6 sections (one for each problem), each with three tables. At each table is a 2-man coordination team. We sit down – the rules are that two of us (Richard and I) are allowed to speak, but Adam may not, although we are allowed to chat to him privately during the coordination.

We quickly agree on 0 for Rebecca, 1 for Marcus, 2 for Oliver and 2 for Jeremy. And Ollie's mark will be 6: two points are given in the mark-scheme for the case of equality, but it is so transparent from his proof what these cases are that he only loses 1 point for not writing them down explicitly. This leaves Thomas. We explain his mistake, and ask for 5. The Coordinators say that he should get 4 out of 5 for his solution, and then 0 out of 2 for not doing the equality case. We say that this seems harsh, as his mistake meant that he *thought* he had not finished the question, and so had not even tried to look for equality, whereas without his slip he would obviously have got the case of equality straight from his proof. The Coordinators say that yes, they understand, but the mark-scheme clearly means that they cannot give more than 4. We say that we agree if the mark-scheme is strictly additive (the two parts being separate), but do not see why it should be. After a little more discussion, we are getting nowhere when suddenly along comes the Senior Coordinator for the question. We explain the situation, and he says 'It seems unfair that he loses 3 points all for one moment of silliness. It deserves 5'. So we go away happy.

We barely have time to recover before we are off with Problem 1. Jeremy scores 7, so we turn our attention to Thomas. The Coordinators are particularly impressed by Thomas' style – there is an intuitively obvious fact one needs (in a particular method of proof), and they say that his is the first of 200 scripts to have actually proved this fact carefully. They are so pleased that they are happy to give him 7 without scrutinising the rest of his answer, so that the argument about 'is this proof worth 6 or 7?', which we had been all set for, never happens. Marcus, Oliver and Rebecca get 6, 5 and 4 respectively, when those could easily have been 5, 4 and 3. So we remain happy.

By now we feel we are experts at coordination sessions. Problem 4 is coordinated,

14

with no battles or surprises. As the day draws to a close, attention begins to focus on the hall where results-so-far are being displayed. Nobody has any firm predictions about medal cutoffs, but the general opinion is that the Gold cutoff will be around 26-30, the Silver around 21 or 22, and the Bronze around 11-13. We do some rapid calculations, based on what we think will happen with the remaining three problems, and it is starting to look more as though we will get four or five Bronzes. It looks like Jeremy, Thomas and Ollie will all just miss out on Silver, with Rebecca getting a clear Bronze and Marcus getting a Bronze if the coordination for Problem 5 is OK and the cutoff is not too high. It is also looking as though Oliver will just miss out on a Bronze. So every point will be vital tomorrow.

July 19: Our day starts early, with our coordination slot for Problem 6. The tension is building. We think we will get four 1s, with 0 for Marcus and either 1 or 2 for Thomas. The Coordinators agree with the four 1s. Then they look at Marcus' script. We say '0, of course', but they point out that, buried among the few lines he has written, is one useful observation, so they give him 1. This was the only time in coordination that the Coordinators spotted something that we had not!

Finally, on to Thomas' script. We point out that he has actually made two, if not three, of the observations that on the mark-scheme appear to be one mark each. The Coordinators say 'Yes, but 1+1+1=1'. They mean that one should not be able to accumulate credit for lots of *little* observations, so that a maximum of 1 mark is available for such things. In fact, this is clearly completely sensible: in a hard question, one wants to be strict, to maximise the benefit to those who *have* solved it. So we quickly agree.

Next comes the coordination for Problem 5. Richard and Adam are in charge of this question: it is they who know the six scripts inside-out, so it is they who will do the talking and arguing. The key scripts are Ollie and Marcus, and these are the last two we deal with. Richard explains why Marcus deserves 5, and the Coordinators explain why they are only offering 4. We decide to take a postponement, as our half-hour is up – we arrange to meet in half an hour to continue. Adam and Richard review Marcus's script feverishly for these 30 minutes, and when we go back in Richard argues very persuasively for 5 marks. The Coordinators want to bring in Ollie's script as well, saying 'well, these scripts are about the

same, and clearly UNK4 deserves 4, so UNK3 deserves 4 as well' (UNK4 and UNK3 being Ollie and Marcus, respectively). But Richard points out some differences, concentrates on Marcus' script, and eventually manages to persuade them that it is worth a 5. The Coordinators say 'OK, our final offer is 5 for UNK3 and 4 for UNK4'. Richard adopts an extraordinary facial expression, completely blank, neither agreeing nor disagreeing – more or less just by his expression he is inviting the Coordinators to go on talking. Since they had been very keen on treating the two scripts together, they talk themselves into offering 5 for Ollie! A triumph for Richard's poker face.

Now there is only Problem 3 to go. We are expecting probably 6 for Rebecca, 1 for Thomas, 1 for Ollie, 0 or 1 for Oliver, 0 for Marcus and 1 or 2 for Jeremy. The smart money is now on the medal cutoffs being 28 or 29, 20 or 21 and 12 or 13. ('Smart money' refers to those people, principally the Irish Deputy, who write down in their little books every single result as it appears in the hall. But accurate predictions are hard, as the best teams tend to have their results appearing slightly later – postponements usually happen over very long and complicated solutions, and of course these are more likely to come from a good team than a bad one).

So it looks as though Marcus and Rebecca are safe Bronzes, Ollie has made a Silver, and Thomas will miss a Silver by 1 or 2 points (there is no way his solution for Problem 3 is worth more than 1 point, unfortunately). And it appears that Jeremy's Silver chances will depend on the cutoff and on his getting 2 rather than 1 for this Problem. So when we go in for Coordination on Problem 3 we are even more nervous than before.

We deal with Thomas, Ollie, Oliver and Marcus. We are ready to fight about Rebecca's solution, in case they try to take 2 points off for her little omission (which would be very unfair), but they immediately say 'Oh yes, this script. Beautiful, beautiful solution, not written down completely correctly, 6 points'. So again the Coordinators show that they have carefully read the scripts beforehand! On to Jeremy. They offer 1 point. We say we want 2, but they point at the mark-scheme and ask us what justifies a second point. The trouble is that there is one particular fact which, although from his *diagram* Jeremy is obviously aware of, he has not written down explicitly. The Coordinators say that they cannot give points for things that are not written down explicitly. But this fact, if it

had been written down, is clearly superior to one of the things that earns 2 points in the mark-scheme. So we say that this superiority means that we ought to get 2 points. After a bit of discussion, they agree.

Meanwhile, things have been happening on the scoreboard. The cutoff for Silver seems to be moving towards 19. We watch excitedly, crowding round the Irish Deputy. Eventually, he is sure it will be 19. We are delighted. So is the Japanese Leader: the Japanese team has three students on 19 (as well as one student on 28, the exact Gold cutoff)!

The Jury quickly meet to ratify the cutoffs, and then it is off to a barbecue with the contestants. Some news has filtered through to them, but not all, so there is much excitement.

July 20: Today we have an all-day trip to Bran Castle ('Dracula's Castle') in the Carpathians. Vlad the Impaler never lived there, but he did attack it once. A strange contrast: from the outside, it looks very Dracula-esque, imposing and sinister, but once inside it is rather cosy and charming. For the Leaders, the coach journey is over familiar territory. Adam, Richard and myself spend the time chatting to the team. Richard gets spectacularly lost on the 50-yard journey from café to coach, causing a 15-minute delay and much amusement.

July 21: We spend the morning shopping or just lazing around. The team come to visit us, and we sit by the pool. Then, cultural experts that they are, they go off to McDonalds for lunch.

In the afternoon is the Closing Ceremony. Yet more speeches. Medals are presented, with much applause. Then off to a restaurant. We all sit outdoors (more than 750 of us – Leaders, Deputies, Observers, contestants, guides and local organisers). It is an extremely friendly occasion.

July 22: Our departure day. We take a taxi to the student accommodation, expecting to meet the team there and take a bus straight to the airport. But not only is there no bus waiting, there is no team waiting either. Richard and Adam rush off to try to find them, while I chat to the taxi driver. It turns out that he knows all about the IMO (as does

everyone in Romania, it seems). Richard and Adam return empty-handed, so we decide that the team must have left for the airport themselves. Off we go in the taxi. We tell the driver that we are in a hurry. The slip road onto the motorway is closed, so he crosses over, goes the wrong way up the down slip-road, and does a U-turn on the motorway, while lorries and buses screech to a halt.

We get to the airport in one piece, and find the team waiting there. We say goodbye to our team's excellent guide, Andrei, and off we fly.

IMO99 was a fantastic experience, very smoothly organised by an army of Romanian mathematicians and other helpers. We are extremely grateful to all of them for the whole wonderful event.

Closer to home, I would like to thank
- all the pupils who took part in any stage of the UK competition
- all the teachers who encouraged them
- Peter Neumann, chairman of the UKMT, and Adam McBride, chairman of the BMOC
- Alan West and Brian Wilson, organisers of BMO1 and BMO2 respectively
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- Christine Farmer for help in the hectic period between BMO2 and the Training Session
- all those who helped with the Trinity Training Session, particularly Julia Gog
- all of our sponsors, particularly Trinity College, Cambridge for hosting the Training Session and the Royal Society for hosting our September celebration
- the DfEE for a grant covering travel to and from the IMO
- Ben Meisner for producing the 1999 BMO booklet, copies of which are now owned by practically every Leader and Deputy
- Richard Atkins and Adam McBride for assistance with the Correspondence Course and for all their help in Romania.

Perhaps most of all, I would like to thank Tony Gardiner and Adam McBride, for telling me so much about the role of Leader and for responding so kindly and helpfully to my millions of worried questions.

This just leaves the squad of 8. Their enthusiasm, energy and sheer intellectual ability has been remarkable. Time and again, all 8 of them surprised me in the Correspondence Course with brilliant or unusual solutions. It has been a pleasure to work with them. Stephen Brooks and Michael Spencer will still be around next year, no doubt fighting for a place in the team, as will Thomas Barnet-Lamb and Oliver Thomas.

The other four are all off to Cambridge this October. Jeremy Young and Rebecca Palmer will study Maths at Trinity College, while Marcus Roper will study Maths at Christ's College. Oliver Wicker will study Music, at King's College. We wish them all the very best for the future.