34th INTERNATIONAL MATHEMATICAL OLYMPIAD
ISTANBUL, TURKEY
13th-24th JULY 1993
Report by ADAM McBRIDE (UK Team Leader)

In preparing this chronicle of events leading up to and during this year's International Mathematical Olympiad (IMO) I have been influenced by the favourable response to last year's report by Tony Gardiner. Although Tony's style is inimitable, I have tried to retain his informal approach in what follows.

Mathematical Competitions

Before we proceed to the main business, a little evangelism is in order. In UK schools we have many youngsters with great mathematical potential. Some go on to realise their full potential but, alas, there are lots who do not. At the age of 12 or 13, bright pupils are thirsting for knowledge and relish a challenge. If these pupils are not stretched mathematically, their fertile minds may turn to other things and be lost to Mathematics. Hard-pressed teachers may not have time to generate "extension material" for their abler pupils but can certainly make use of material generated by others. It is here that mathematical competitions come in. There is now a range of such competitions covering all years in secondary schools. Yet the existence of such competitions is not well known to many teachers and even less well known to pupils.

Most problems set in competitions require relatively little mathematical knowledge and so can often be solved by using only a little flair or insight. Yet they are far from "easy" and teach pupils that not all problems can be solved in five minutes flat. The patience and determination developed through tackling problems are essential attributes for success in life in general. Since we shall be talking mainly about Olympiads, let us remember the old ideal that taking part is more important than winning. We can't all be winners but all can learn from the experience; many who struggle first time round respond in the intended way by setting their sights that much higher the next time.

In Australia 92% of high schools take part each year in the Australian Mathematics Competition. We have a long way to go before we can match this figure but my message to all those reading this document is

SPREAD THE WORD.

Those already involved should contact colleagues in neighbouring schools or regions and whip up support. As a Scot, I am particularly keen to see increased participation from schools in Scotland, Wales and Northern Ireland. A small amount of effort could produce considerable rewards. Go to it!

A number of competitions will be mentioned below, most of them falling under the aegis of the British Mathematical Olympiad Committee (BMOC). For further information on these and other competitions contact the BMOC Secretary who is

Dr. Tony Gardiner, School of Mathematics and Statistics
University of Birmingham, BIRMINGHAM B15 2TT

After that clarion call, we can turn to the events leading up to the 1993 IMO.

Selecting the UK Team

The road to Istanbul was a long one. The first stage was the National Mathematics Contest (NMC), run by the Mathematical Association and held in November 1992. It attracted around 30000 entrants. This was a substantial increase on the 1991 entry but we hope the 1993 entry will be even larger. The NMC paper contains 25 multiple choice questions; the first 15 or so are accessible to most contestants while the last few problems are more taxing. Based largely on their performances in the NMC, over 600 pupils entered the British Mathematical Olympiad Round 1 (BMO 1), a $3\frac{1}{2}$-hour paper with 5 questions held in January. Almost 90 pupils were then selected to tackle BMO 2, another $3\frac{1}{2}$-hour paper but with just 4 questions and held in February. The selection for BMO 2 contained a number of "wild cards" in the form of pupils whose BMO 1 marks may have been disappointing but who nevertheless showed promise. Most of these selections were duly vindicated and some of the younger contestants will have derived great benefit from the experience. We look forward to seeing what they achieve this coming year. The BMO 2 field then had to be reduced to exactly 20 for the residential Training Session at Easter. This proved a tricky business, especially as we always try to include 8 to 10 younger students.

The chosen 20 gathered at Trinity College, Cambridge for some systematic instruction during the period 15th-18th April. For three days, they had 4 or 5 intensive 2-hour sessions each day, mostly spent on actually tackling problems rather than on formal lectures, covering Algebra, Combinatorics, Geometry, Inequalities and Number Theory. They also received advice on how to tackle hard problems and how to write out solutions. Light relief was provided by a relay competition in which a team of former UK IMO team members won narrowly, despite a spritely performance from the Wrinklies (a distinguished subset of the instructors), and some excellent work from the five student teams. After surviving this mathematical onslaught, our young hopefuls were rewarded with a $4\frac{1}{2}$-hour examination called the Final Selection Test (FST) on the last morning. This was intended to simulate a full-blooded IMO paper and so contained just three questions.

The Training Session ended with the presentation of awards to all 20 students by Professor Sir Michael Atiyah, Master of Trinity College and President of BMOC. When the students had all left Cambridge, we were left with the job of making the final selection, which again was far from easy.

UK Team for 34th IMO

After the Training Session, the UK team for the 1993 IMO was selected on the basis of all the information then before us. The team was as follows:

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Tom Fisher (Exeter College)
Alistair Flutter (Hills Road Sixth Form College, Cambridge)
Catriona MacLean (Harrogate Grammar School)
Alex Paseau (St. Paul's School, London)
Luke Pebody (Rugby School)
Chuan Tze Teo (Dulwich College, London)

Reserve: Paul Hacking (Lancaster Royal Grammar School)
Team Leader: Adam McBride (University of Strathclyde, Glasgow)
Deputy Leader: Christopher Bradley (Clifton College, Bristol)

Those in their last year at school who attended the Training Session but who were not selected will have their chance at university to show what they can do. The younger students who will be eligible for consideration again next year are presently being kept ticking over by a correspondence course kindly run by David Monk.

Final Preparations

During May and June the team embarked on a correspondence course with a set of problems arriving every ten days. Of course, some already had plenty of other examinations to prepare for but they survived. Our reserve, Paul Hacking, deserves special mention for setting the pace throughout the correspondence course; he would have been a worthy replacement should any of the team have suffered a mishap. From 3rd to 5th July, the team plus Christopher Bradley and myself gathered for a weekend in Birmingham. The activities were deliberately less structured than in Cambridge. For example, we all tried some problems from current issues of some journals including Crux Mathematicorum and the American Mathematical Monthly. We wrote up solutions to a couple of problems together and sent them off in the hope of seeing our collective name in print before too long. This illustrated one of the main objectives of the weekend, namely engendering team spirit and getting to know each other better. We also had to attend to mundane practical matters. Christopher had already issued his "Marching Orders" and the final details of our itinerary were checked.

This gathering formed a very important part of our preparations. That it took place at all and, in particular, in Birmingham was due to the hospitality of Gwyneth Gardiner whose normal household of 7 became a mini-hotel for 15. Fortunately, the weather was fine and some of the younger Gardiners were able to sleep under canvas in the garden. Other countries may have had more money lavished on their final preparations but none could have enjoyed such warm and generous hospitality and such good food as we received. We are all most grateful.

And so to IMO itself.

34th IMO: Some Statistics

A total of 413 contestants from 73 countries took part.

The UK team finished 14th out of 73 with 118 points (out of 252).

Each of the 6 team members won a medal, the haul being

3 Silver Medals and 3 Bronze Medals.

The top four teams were

China 215, Germany 189, Bulgaria 178, Russia 177.

A total of 118 questions were proposed by 43 participating countries.

All 3 UK proposals were short-listed, with one being chosen as question 2 (see below) and a second almost being chosen. The latter two were both composed by David Monk (Edinburgh) and the other UK proposal was by Christopher Bradley.

Question papers were prepared in 47 different languages.

The Questions

All contestants sit two papers on successive days. Each paper contains 3 questions, each question being worth 7 points.

On each day the time allowed was $4\frac{1}{2}$ hours.

The questions were proposed by the countries indicated.

FIRST DAY

1. Let $f(x) = x^n + 5x^{n-1} + 3$ where $n > 1$ is an integer. Prove that $f(x)$ cannot be expressed as the product of two polynomials, each of which has all its coefficients integers and degree at least 1.

(Ireland)

2. Let D be a point inside the acute-angled triangle ABC such that

$$\angle ADB = \angle ACB + 90°$$

and
$$AC \cdot BD = AD \cdot BC.$$

(a) Calculate the value of the ratio $\dfrac{AB \cdot CD}{AC \cdot BD}$.

(b) Prove that the tangents at C to the circumcircles of the triangles ACD and BCD are perpendicular.

(United Kingdom)

3. On an infinite chessboard, a game is played as follows.

At the start, n^2 pieces are arranged on the chessboard in an $n \times n$ block of adjoining squares, one piece in each square. A move in the game is a jump in a horizontal or vertical direction over an adjacent occupied square to an unoccupied square immediately beyond. The piece which has been jumped over is then removed.

Find those values of n for which the game can end with only one piece remaining on the board.

(Finland)

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SECOND DAY

4. For three points P, Q, R in the plane, we define $m(PQR)$ to be the minimum of the lengths of the altitudes of the triangle PQR (where $m(PQR) = 0$ when P, Q, R are collinear).

   Let A, B, C be given points in the plane. Prove that, for any point X in the plane,

   $$m(ABC) \leq m(ABX) + m(AXC) + m(XBC).$$

   <div align="right">(Macedonia)</div>

5. Let $\mathbf{N} = \{1, 2, 3, ...\}$.

   Determine whether or not there exists a function $f : \mathbf{N} \to \mathbf{N}$ such that

   $$f(1) = 2,$$
   $$f(f(n)) = f(n) + n \quad \text{for all } n \in \mathbf{N}$$
   $$\text{and} \quad f(n) < f(n+1) \quad \text{for all } n \in \mathbf{N}.$$

   <div align="right">(Germany)</div>

6. Let $n > 1$ be an integer. There are n lamps $L_0, L_1, ..., L_{n-1}$ arranged in a circle. Each lamp is either ON or OFF. A sequence of steps $S_0, S_1, ..., S_i, ...$ is carried out. Step S_j affects the state of L_j only (leaving the state of all other lamps unaltered) as follows:

   if L_{j-1} is ON, S_j changes the state of L_j from ON to OFF or from OFF to ON;

   if L_{j-1} is OFF, S_j leaves the state of L_j unchanged.

   The lamps are labelled mod n, that is

   $$L_{-1} = L_{n-1} \quad , \quad L_0 = L_n \quad , \quad L_1 = L_{n+1}, \quad etc.$$

   Initially all lamps are ON.

   Show that

   (a) there is a positive integer $M(n)$ such that after $M(n)$ steps all the lamps are ON again;

   (b) if n has the form 2^k then all the lamps are ON after $n^2 - 1$ steps;

   (c) if n has the form $2^k + 1$ then all the lamps are ON after $n^2 - n + 1$ steps.

   <div align="right">(The Netherlands)</div>

How the UK Team Performed

	Q1	Q2	Q3	Q4	Q5	Q6	Total	Medal
Tom Fisher	1	1	7	5	7	7	28	Silver
Alistair Flutter	7	0	1	1	6	0	15	Bronze
Catriona MacLean	7	0	7	0	6	0	20	Silver
Alex Paseau	7	0	1	2	7	0	17	Bronze
Luke Pebody	1	4	6	4	7	5	27	Silver
Chuan Tze Teo	1	0	4	3	2	1	11	Bronze
Total	24	5	26	15	35	13	118	

It is tragic to see that our worst performance by far was in the geometry question that the UK had proposed! Students (and teachers) who wish to take Mathematics seriously must make greater efforts to come to terms with basic geometry. (A series of four booklets to help them do just this is available at a cost of £2 each from Tony Gardiner at the above address; make cheques payable to "BMOC".)

Overall Performance of All Contestants

When the questions were finally selected, many of us felt that we were setting the contestants a very stiff challenge. This was confirmed by the marks. Only 2 contestants (one each from China and Taiwan) scored the maximum of 42. A contestant from Germany scored 39. The next best score was 37. The range of scores for the various medals was as follows:

Gold	From	31	to	42	(35 contestants)
Silver	From	20	to	30	(66 contestants)
Bronze	From	11	to	19	(97 contestants).

Only 12 teams scored at least half marks (≥ 126). There was a big gap between Canada and Israel who were 18th equal with 113 and Japan who were 20th with 98. Some countries did not field a full team of six. (It is worth mentioning that the IMO is meant to be a competition for individuals not teams. However, more attention is usually paid to team performances.)

How the 34th IMO was run

The organisation was superb and no expense was spared in making sure that everything went as planned. (It is perhaps not irrelevant to remark that Istanbul is one of the cities involved in the final contest to host the other Olympics in the year 2000.) Team Leaders, Deputy Leaders and Teams all received some spending money (a few hundred thousand Turkish liras!), an IMO T-shirt, badge, posters and a shoulder bag (for the Teams) or canvas brief case (Leaders and Deputy Leaders). A full programme of excursions was arranged to enable us to make the best use of what spare time we had. (See the diary below.)

Transporting 800 people from A to B requires an operation of military precision, particularly so when most of the 800 are mathematicians! Each team was allocated a

guide, usually a local student who could speak the appropriate language. The guides made sure that the teams were at the right place at the right time. Events elsewhere in Turkey meant that security was an important matter. A security person travelled on every bus and escorted us on excursions. (This also helped to make sure that there was no contact between Team Leaders and their Teams/Deputy Leaders until after the end of the second paper.) The cavalcade of 20 buses made an impressive sight as it swept along with a police car at the head and an ambulance at the rear. The latter was a sensible idea. If you take a large group of adolescents anywhere, there is a sporting chance that someone may suffer cuts, bruises or a sprained ankle. Fortunately, the ambulance did not seem to be needed, although 10 contestants were sick after eating some water-melon.

All decisions concerning the competition were in the hands of the Jury which consisted of the 73 Team Leaders, each of whom had one vote. We were each given a red plastic wand with our own 3-letter code marked on it. The UK code was UNK so that I became "The Man From UNK". Votes were frequent and consisted of the chairman counting the number of raised wands. Jury meetings were like a scaled down version of the House of Commons. On occasions, the whole thing threatened to turn into a bear garden as an argument broke out, usually in a foreign language, among some of the more volatile Leaders. The business of the Jury was conducted principally in English but interpreters were on hand to provide translations into French and Russian. (German did not seem to be required.) Our main tasks were to select the problems to be set, approve the various translations, confirm the marks scored, award medals and deal with any other matter related to the IMO regulations. That we succeeded in achieving this was a great credit to the person who chaired the majority of our meetings. Professor Semih Koray earned the respect of everyone by conducting the business firmly but with charm and tact.

The other major task for the Leaders and Deputy Leaders was the marking of the scripts. We each marked the scripts of our own Team and then had to justify our marking before a panel of Co-ordinators. A schedule was drawn up whereby each country was allocated 6 half-hour slots over a 3-day period. In each slot the marking of one specified question was discussed. Inevitably, some difficulties arose and the schedule was knocked well off course. Some teams were still going strong (?) at 3 o'clock in the morning when the session should have finished at 22.30. Fortunately Christopher Bradley and I were not affected by any major delays and we felt that the Co-ordinators had been tough but fair. Here again I should like to single out one person for special mention. Professor Albert Erkip was responsible for co-ordinating the Co-ordinators as well as chairing the group which short-listed the problems for the Jury. He also superintended the production of the 47 translations and the posting of marks on the display boards as the drama unfolded. All of these tasks he discharged with remarkable sang-froid and good humour. For me, Professor Koray and Professor Erkip were two people who played a huge part in making the 34th IMO such a success.

To give a flavour of what really happened I offer a

Diary of The Man From UNK

Tuesday 13th

Arrive in Istanbul at 18.00. Taken to the hotel which is to be our base for the next 5 days. Very posh, very American. After dinner, first Jury meeting at 21.00 is brief and we all go off to tackle the short-listed problems (without peeping at the solutions).

Wednesday 14th

In the hotel all day, with Jury sessions at regular intervals. As time passes, the short list gets shorter and shorter.

Thursday 15th

We get down to 4 possible (non-disjoint!) sets of 6 questions and these are eliminated one by one. In 2 of the 4, there are 2 questions proposed by the UK but the final choice has just one UK question. I find myself in charge of the group preparing the official English versions of the questions. Those involving the chessboard and the lamps could be tricky. Taken to an evening concert of music for chamber choir. The choir scores highly for effort but a choral arrangement of "Ich grolle nicht" from Schumann's *Dichterliebe* is less successful (to be rivalled only by the UK team's performance on Question 2!). Work until 02.00 to get the lamp problem licked into shape.

Friday 16th

Official versions of the problems in English, French, German, Russian and Spanish approved before lunch. Rest of the day highly frustrating as Leaders prepare the other 42 translations. Modern technology is wonderful if it works but the software fails to cope. Many Leaders give up and write things out by hand. Many errors to be eradicated. Final approval of all versions only possible after midnight. Meanwhile Christopher and the Team will have arrived, I hope!

Saturday 17th

Opening Ceremony at 10.00. Leaders arrive at 09.00 and are shepherded into the gallery. Teams and Deputy Leaders arrive a little later and are put in the stalls. Leaders know the questions; so we are held incommunicado. Spot Christopher and the team with distinctive T-shirts (courtesy of Luke and family). My T-shirt is delivered by air mail (thrown up to the gallery from below). Ceremony starts with a curious modern ballet. Learn later that it had something to do with telephone chatlines. A few speeches including one by the Deputy Prime Minister which I fail to understand because someone has pinched my headphones (for simultaneous translation). Best speech is by a young university student who competed in IMO in 1991. Leaders taken off on excursion until dinner. Visit the "Blue Mosque", Aya Sofya and Topkapi Palace. Get first real glimpse of the Bosphorus from Topkapi. Scenery stunning. Can't wait for tomorrow's boat trip.

Sunday 18th

First paper from 09.00 to 13.30. During first half-hour, contestants may send written

queries to the Jury if they need any clarification about what a question means. Queries are brought to the Jury by runner from the exam hall next door and are answered in order of receipt. Some get a dusty response such as "No comment" or "Read the question" but others are treated more sympathetically where it is felt appropriate. Only 30 questions received, rather fewer than usual. The efforts expended on preparing the official versions have not been in vain. Leaders leave to take part in the "Parade of Boats" marking the end of a festival of history and the sea. Five little puffers have been decorated with IMO banners. Sail up towards the Black Sea and then back down in procession. Glorious afternoon, nice breeze to temper the heat, magnificent scenery. Back to base to collect scripts from the morning exam. Stay up until 1 a.m. marking. Disaster on question 2!

Monday 19th

Leaders moving to join Deputy Leaders after second paper. Teams will be close by. Jury receives 52 queries. Then bad news! During the first paper, one contestant was found to have a pocket book of formulae in the exam hall. Subgroup set up to ascertain the facts. Full Jury meeting called for 22.30.

Meet team at end of exam. They feel that they have done quite well on the second day, especially in question 5, but are apprehensive about the first day. Meet up with Christopher who has been on an excursion. Examine co-ordination schedule. Foresee a very long night ahead.

Scripts for second paper arrive around 18.30. We must have the lamp question sorted out completely by 09.00 the next morning.

First co-ordination at 21.00. This is the chessboard question and goes well. (We did a lot of chessboard problems during training, though none quite like this.) Then at 22.00, Christopher and I change sides and become co-ordinators of the Turkish responses to our UK geometry question. Running behind schedule now and I miss start of Jury meeting. Contestant deemed to have cheated and is disqualified. Worse is to follow! On the second day two contestants were found in possession of calculators in the exam hall, contrary to the rules. Another subgroup set up to investigate. Back to room at midnight to deal with the lamp question. Spend 3 hours with Christopher trying to unscramble Luke's solution with partial success. Bed at 04.00.

Tuesday 20th

Four co-ordinations throughout the day. We get 5 out of 7 for Luke's lamps. Burning the midnight oil was worth it. Marks start to appear on the display boards. By 22.30 we have a fair idea of where we shall finish, somewhere between 12th and 17th. The worst of our labours seem to be over.

Wednesday 21st

Last co-ordination over by 09.30. Spend morning watching final pieces of jig-saw appearing on display boards. Spend afternoon in downtown Istanbul, including a visit to the Covered Bazaar. Run the gauntlet of people trying to sell us perfume, jewellery, leather jackets, carpets and much more.

Final Jury meeting starts at 21.00. Two contestants with calculators are awarded 0 points for the second day. Marks for all students approved and distribution of medals agreed. Then trouble! Allegations that one question had been used in an earlier competition and could have been familiar to some contestants. Chairman calls a short break to allow things to calm down. Some contestants have arrived to find out the results. Meeting resumes at 23.30. Report from Site Committee (in future to be called Advisory Board) which keeps things on the move between IMO's. Host countries for future years are noted and approved (up to 1998). Discussion on method of selecting problems and possible changes to regulations. A few more votes and we end at 01.10.

Thursday 22nd

Quiet morning. Send a FAX to Tony containing the main statistics. Then we are all at sea. About 800 people pile aboard a huge ferry for an 8-hour cruise on the Sea of Marmara and the Bosphorus. We visit the Princes' Islands and then stop to take on provisions. Now it really is party time. A disco is in full swing on the middle deck with the young (and some of the not so young) from all over the world dancing and singing. I spend most of the time at the stern watching this amazing sight while soaking up a glorious sunset. The scenery we saw on Sunday in bright sunshine is just as beautiful in the dusk. Bus journey back very quiet. Most people have crashed out.

Friday 23rd

The team, Christopher, John Hersee (Secretary of the Site Committee) and myself pay an early morning courtesy call to the British Council and meet the Consul (Cultural) Istanbul who has taken a keen interest in our visit. Hail 3 taxis to get us back to the hotel. The 3 drivers seem to be pals and decide to have a race. All driving in Istanbul is crazy but this taxi journey is horrendous! Closing Ceremony starts at 14.30 with some enjoyable singing and dancing. Then comes the prize-giving. Specially loud cheers for Turkish medal-winners and a girl from Bosnia. The Croatians prove to be a lively bunch and Katy's hat takes a trick. Various speeches follow with everybody congratulating everybody else. It is Professor Koray who epitomises the whole spirit of the occasion by reminding us that

MATHEMATICS IS A UNIVERSAL LANGUAGE

uniting all countries of the world. Then as the lights dim, all the guides and other helpers march in with luminous sticks. Everyone sings "We are the world, we are the children" and we all vow to meet again in Hong Kong in 1994. It would be a strange person indeed who could fail to find this occasion intensely emotional.

Time to relax and celebrate informally. We cross the Bosphorus and I set foot in Asia for the first time (and have a certificate to prove it). We go to a forest which forms the highest part of Istanbul. Party time again! Take a picture of our team together with representatives of Canada, Colombia, Croatia, Ireland, New Zealand and Slovakia. Discover a Hungarian who has already managed to lose his gold medal! A wedding party appears. I bet they got a big surprise to find not teddy bears having a picnic but 800 mathematicians. All the while young entrepreneurs aged about 5 invite us to buy

postcards and chewing gum.

Finally the cavalcade makes the short trip to Beylerbeyi Palace for a banquet. No formal speeches, just everyone dining alfresco under the magnolias beside the Bosphorus with an orchestra playing. By this time Alistair has lost and recovered just about everything and the Hungarian has been reunited with his gold medal. So everyone is happy.

Saturday 24th

Team roused at 06.10 by their guide, the indefatigable Alp whose great efficiency and cheerfulness were much appreciated by all of us, especially by Christopher. Everybody gets to the airport safely. Most of the team sleep on the flight home. Katy finds the room key lost several days earlier. Arrive on time at Heathrow to be met by parents.

Concluding Remarks

The 34th IMO was an unforgettable experience for all those lucky enough to be there. Not everyone was a winner but at the disco or in the forest nobody was worried. Young people from all over the world (including some of its most troubled regions) were enjoying themselves, exchanging addresses, signing T-shirts and making friendships which may last throughout their lives. The universal language of Mathematics was the catalyst that brought this about. It was a great honour and enormous pleasure to lead the UK team and I congratulate them on their achievement.

Our thoughts now turn to the 35th IMO to be held in Hong Kong during the period 8th-20th July 1994. Soon the process of picking the UK team will begin. Along the way there will be winners and losers. However, we should do all we can to enable as many talented youngsters as possible to experience the unique character of this universal language of Mathematics. I encourage all of you to read again my initial remarks about Mathematical Competitions. Since we now seem to be back where we started, it is time to stop.

Acknowledgements